Atomic Data for Ti Ions

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Applications

1. Astrophysical Plasmas
2. Solar Plasmas
3. Lasing Plasmas
4. Fusion Plasmas

Atomic Parameters

- Energy Levels

$$E_j - E_i = h\nu_{ij} = hc/\lambda_{ij}$$

- Radiative Rates (A, s^{-1}),
  Oscillator Strengths (f, dimensionless),
  Line Strengths (S, a.u.)

$$f_{i,j} = \frac{mc}{8\pi^2 e^2}\lambda_{ji}^2 \frac{\omega_j}{\omega_i} A_{ji} = 1.49 \times 10^{-16}\ \lambda_{ij}^2(\omega_j/\omega_i)A_{ji}$$

$$\text{E1:} \quad A_{ji} = \frac{2.0261\times10^{18}}{\omega_j\lambda_{ji}^3} S \quad \text{and} \quad f_{ij} = \frac{303.75}{\lambda_{ji}\omega_i} S,$$

$$\text{E2:} \quad A_{ji} = \frac{1.1199\times10^{18}}{\omega_j\lambda_{ji}^5} S \quad \text{and} \quad f_{ij} = \frac{167.89}{\lambda_{ji}^3\omega_i} S,$$

$$\text{M1:} \quad A_{ji} = \frac{2.6974\times10^{13}}{\omega_j\lambda_{ji}^3} S \quad \text{and} \quad f_{ij} = \frac{4.044\times10^{-3}}{\lambda_{ji}\omega_i} S,$$

$$\text{M2:} \quad A_{ji} = \frac{1.4910\times10^{13}}{\omega_j\lambda_{ji}^5} S \quad \text{and} \quad f_{ij} = \frac{2.236\times10^{-3}}{\lambda_{ji}^3\omega_i} S.$$

λ is in Å.

Atomic Parameters

- Life-Time

$$\tau_j = \frac{1}{\sum_i A_{ji}}$$

- Collision Strengths (Cross Sections)

$$\Omega_{ij}(E) = k_i^2 \omega_i \sigma_{ij} (\pi a_0^2)$$

- Effective Collision Strengths (Rate Coefficients)

$$\Upsilon(T_e) = \int_0^\infty \Omega e^{-E_j/kT_e} d(E_j/kT_e)$$

$$q_{ij} = \frac{8.63\times10^{-6}}{\omega_i T_e^{1/2}} e^{-E_{ij}/kT_e} \Upsilon_{ij} \qquad \mathrm{cm^3/s}$$

$$q_{ji} = \frac{8.63\times10^{-6}}{\omega_j T_e^{1/2}} \Upsilon_{ij} \qquad \mathrm{cm^3/s}$$

- Line Intensity Ratio

$$\mathrm{I}_{ji} = A_{ji} N_j N_{A,Z} N_A h\nu_{ji} \frac{n}{1+N_{He}} \frac{L}{4\pi} \qquad \mathrm{ergs\ cm^{-2}\ s^{-1}\ sr^{-1}}$$

$$\mathrm{R} = \frac{I(\lambda_{ij})}{I(\lambda_{mn})} = \frac{A_{ji}}{A_{nm}} \frac{\lambda_{mn}}{\lambda_{ij}} \frac{N_j}{N_n}$$

Cl-LIKE Ti VI

Energy levels, A-values and Lifetimes
568 levels among the $n \leq 4$ configurations

Al-LIKE Ti X

Energy levels, A-values and Lifetimes
628 levels among the $n \leq 4$ configurations

Be-LIKE Ti XIX

Energy levels, A-values and Lifetimes,
Collision Strengths (Ω) and Excitation Rates (Υ)
98 levels among the $n \leq 4$ configurations

Phys Scr **86** (2012) 000000

Li-LIKE Ti XX

Energy levels, A-values and Lifetimes,
Collision Strengths (Ω) and Excitation Rates (Υ)
24 levels among the $n \leq 5$ configurations

ADNDT **98** (2012) 000

He-LIKE Ti XXI

Energy levels, A-values and Lifetimes,
Collision Strengths (Ω) and Excitation Rates (Υ)
49 levels among the $n \leq 5$ configurations

Phys Scr **85** (2012) 065301

Programs

1. **GRASP0**
   PH Norrington
   `http://web.am.qub.ac.uk/DARC/`
2. **DARC**
   PH Norrington & IP Grant
   `http://web.am.qub.ac.uk/DARC/`
3. **FAC**
   MF Gu, Can J. Phys. **86** (2008) 675
   `http://sprg.ssl.berkeley.edu/~mfgu/fac/`

Table 1. Target levels of Ti VI and their threshold energies (in Ryd). – Part 1

Index	Configuration	Level	NIST	GRASP1	GRASP5	GRASP7	FAC	MCHF	CIV3
1	$3s^23p^5$	$^2P^o_{3/2}$	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
2	$3s^23p^5$	$^2P^o_{1/2}$	0.05312	0.05349	0.05253	0.05246	0.05190	0.04614	0.05312
3	$3s3p^6$	$^2S_{1/2}$	1.79181	1.70921	1.80736	1.77319	1.78286	1.77535	1.79133
4	$3s^23p^4$ (^3P)3d	$^4D_{7/2}$		2.34891	2.50467	2.43448	2.41939		2.41651
5	$3s^23p^4$ (^3P)3d	$^4D_{5/2}$		2.35198	2.50761	2.43741	2.42232	2.42631	2.41925
6	$3s^23p^4$ (^3P)3d	$^4D_{3/2}$		2.35665	2.51215	2.44195	2.42684	2.42994	2.42359
7	$3s^23p^4$ (^3P)3d	$^4D_{1/2}$		2.36066	2.51607	2.44587	2.43074	2.43319	2.42740
8	$3s^23p^4$ (^3P)3d	$^4F_{9/2}$		2.55963	2.73200	2.66201	2.64284		2.63250
9	$3s^23p^4$ (^1D)3d	$^2P_{1/2}$	⋆	2.61445	2.73481	2.68027	2.65491	2.64166	3.67866⋆
10	$3s^23p^4$ (^3P)3d	$^4F_{7/2}$		2.57776	2.74964	2.67976	2.66045		2.65034
11	$3s^23p^4$ (^3P)3d	$^4F_{5/2}$		2.59048	2.76228	2.69224	2.67287	2.65636	2.66286
12	$3s^23p^4$ (^3P)3d	$^4F_{3/2}$		2.59837	2.76649	2.69988	2.68048	2.66283	2.67046
13	$3s^23p^4$ (^1D)3d	$^2P_{3/2}$	⋆	2.64550	2.77021	2.71180	2.68615	2.66997	3.65922⋆
14	$3s^23p^4$ (^3P)3d	$^4P_{1/2}$		2.68638	2.81951	2.77456	2.74822	2.73440	2.73731
15	$3s^23p^4$ (^1D)3d	$^2D_{3/2}$	⋆	2.69733	2.83078	2.77650	2.75251	2.73788	3.79308⋆
16	$3s^23p^4$ (^3P)3d	$^4P_{3/2}$		2.70483	2.84128	2.78719	2.76091	2.74586	2.75149
17	$3s^23p^4$ (^3P)3d	$^4P_{5/2}$		2.71212	2.84537	2.79907	2.77285	2.75765	2.76331
18	$3s^23p^4$ (^1D)3d	$^2D_{5/2}$	⋆	2.73284	2.86894	2.80799	2.78320	2.76541	3.74875⋆

Table 1. Target levels of Ti VI and their threshold energies (in Ryd). – Part 2

Index	Configuration	Level	NIST	GRASP1	GRASP5	GRASP7	FAC	MCHF	CIV3
19	$3s^23p^4$ (^{3}P)3d	$^2F_{7/2}$		2.74745	2.89542	2.83627	2.81121		2.79674
20	$3s^23p^4$ (^{1}D)3d	$^2G_{9/2}$		2.77937	2.95585	2.87315	2.84997		2.83908
21	$3s^23p^4$ (^{1}D)3d	$^2G_{7/2}$		2.78624	2.95562	2.87741	2.85348		2.84163
22	$3s^23p^4$ (^{3}P)3d	$^2F_{5/2}$		2.79677	2.93693	2.88213	2.85591	2.82840	2.84143
23	$3s^23p^4$ (^{1}D)3d	$^2F_{5/2}$		3.03169	3.16197	3.10934	3.07466	3.04348	3.05835
24	$3s^23p^4$ (^{1}D)3d	$^2F_{7/2}$		3.04294	3.17318	3.12061	3.08583		3.07002
25	$3s^23p^4$ (^{1}S)3d	$^2D_{3/2}$		3.27911	3.40778	3.29467	3.26967	3.23478	3.25926
26	$3s^23p^4$ (^{1}S)3d	$^2D_{5/2}$		3.28966	3.41901	3.30421	3.27953	3.24467	3.27015
27	$3s^23p^4$ (^{1}D)3d	$^2S_{1/2}$		3.70763	3.63458	3.54068	3.51941	3.53341	3.58151
28	$3s^23p^4$ (^{3}P)3d	$^2P_{3/2}$	2.65990⋆	3.69560	3.88504	3.69879	3.66809	3.62499	2.67311⋆
29	$3s^23p^4$ (^{3}P)3d	$^2P_{1/2}$	2.62820⋆	3.73072	3.90475	3.71755	3.68665	3.64205	2.64098⋆
30	$3s^23p^4$ (^{3}P)3d	$^2D_{5/2}$	2.75554⋆	3.83147	3.93068	3.75162	3.72195	3.70361	2.77788⋆
31	$3s^23p^4$ (^{3}P)3d	$^2D_{3/2}$	2.72461⋆	3.87593	3.97650	3.79587	3.76592	3.74244	2.74597⋆

NIST (`http://www.physics.nist/gov/PhysRefData`)
GRASP1: Present results from 3 configurations and **60** levels
GRASP5: Present results from 16 configurations and **568** levels
GRASP7: Present results from 37 configurations and **4032** levels
FAC: Present results with **5821** levels
MCHF: Results of Forese-Fischer et al, ADNDT **92** (2006) 607
CIV3: Results of Mohan et al, ADNDT **93** (2007) 105

Table 3: Comparison of oscillator strengths for some transitions of Ti VI. ($a{\pm}b \equiv a{\times}10^{\pm b}$). – Part 1

i	j	f (GRASP1)	f (GRASP5)	f (GRASP7)	A (FAC)	MCHF	CIV3
1	3	2.4157−02	3.3264−02	2.5418−02	2.523−02	2.581−2	2.060−2
1	5	5.1087−05	3.4947−05	4.1045−05	3.978−05	2.392−5	3.013−5
1	6	1.5708−05	1.4265−05	1.7708−05	1.691−05	1.260−5	1.477−5
1	7	2.2003−06	2.5338−06	3.2395−06	3.770−06	2.662−6	2.741−6
1	9	2.6142−04	1.6344−04	3.2000−04	2.973−04	2.672−4	2.241−4
1	11	1.6798−04	1.3220−04	1.6249−04	1.686−04	1.104−4	1.311−4
1	12	3.1549−05	1.8518−03⋆	1.4570−05	5.069−06	7.934−6	3.996−7⋆
1	13	2.7064−03	6.8688−05⋆	3.1356−03	2.954−03	2.450−3	2.324−3
1	14	3.0295−04	3.3459−04⋆	3.5759−04	3.606−04	2.520−4	2.870−4
1	15	4.9097−04	2.4324−04⋆	1.4166−03	1.357−03	1.104−3	1.018−3
1	16	9.9916−04	9.1854−04⋆	9.3892−06	2.695−05	1.071−5	2.495−4
1	17	2.4504−04	1.5016−04	9.0254−04	5.835−04	4.455−4	2.806−4
1	18	3.5888−03	2.9121−03	2.8958−03	2.983−03	2.633−3	3.023−3
1	22	8.2587−05	3.7032−05	8.0851−05	6.810−05	5.276−5	6.677−5
1	23	6.3453−04	5.8361−04	6.9690−04	7.158−04	4.749−4	5.675−4
1	25	8.0809−04	5.7437−05⋆	1.0724−03	8.816−04	6.673−4	5.677−4
1	26	3.7004−05	1.6588−03⋆	3.1199−04	7.989−05	3.075−5	5.217−5
1	27	5.8702−01⋆	4.1355−01	4.1759−01	4.169−01	4.577−1	4.652−1
1	28	1.1790+00⋆	1.0599+00	1.0153+00	1.017+00	1.014+0	9.944−1
1	29	2.4254−01⋆	1.6675−01	1.5285−01	1.527−01	1.397−1	1.285−1
1	30	2.4577+00⋆	1.8944+00	1.8339+00	1.829+00	1.910+0	1.878+0
1	31	1.7962−01⋆	7.2171−02	8.0342−02	8.079−02	1.124−1	1.108−1

Table 3: Comparison of oscillator strengths for some transitions of Ti VI. ($a{\pm}b \equiv a{\times}10^{\pm b}$). – Part 2

i	*j*	f (GRASP1)	f (GRASP5)	f (GRASP7)	A (FAC)	MCHF	CIV3
2	3	2.3909−02	3.2605−02	2.5036−02	2.491−02	2.665−2	2.066−2
2	6	1.0501−05	4.5799−06	5.1811−06	4.803−06	2.162−6	2.620−6
2	7	7.3214−06	9.0097−06	1.1639−05	1.169−05	9.259−6	1.069−5
2	9	1.9734−03	1.5570−03	2.4547−03	2.316−03	1.918−3	1.773−3
2	12	2.7412−04	1.4367−03⋆	2.6902−04	3.246−04	1.988−4	3.258−4
2	13	1.8906−03	1.2168−04⋆	2.0645−03	1.896−03	1.564−3	1.418−3
2	14	9.4429−05	1.0954−04⋆	1.0524−04	9.586−05	7.407−5	7.121−5
2	15	9.1736−04	2.2825−04⋆	3.5330−03	3.074−03	2.723−3	1.915−3
2	16	2.7189−03	2.8189−03⋆	3.0878−04	4.637−04	2.797−4	1.332−3
2	25	4.2760−03	7.7370−03	1.7263−03	2.521−03	2.070−3	3.771−3
2	27	2.3707−01⋆	3.5330−01	3.3195−01	3.290−01	3.192−1	3.035−1
2	28	2.5081−01⋆	1.0220−01	1.1833−01	1.205−01	1.713−1	1.674−1
2	29	1.2313+00⋆	7.7024−01	7.6450−01	7.670−01	8.183−1	8.186−1
2	31	2.9086+00⋆	2.3609+00	2.2700+00	2.261+00	2.312+0	2.273+0

GRASP1: Present results from 3 configurations and **60** levels
GRASP5: Present results from 16 configurations and **568** levels
GRASP7: Present results from 37 configurations and **4032** levels
FAC: Present results with **5821** levels
MCHF: Results of Forese-Fischer et al, ADNDT **92** (2006) 607
CIV3: Results of Mohan et al, ADNDT **93** (2007) 105

CI is very important even for strong transitions and results are highly variable for weak transitions

Table 1. Energies (Ryd) for the lowest 40 levels of Ti X. – Part 1

Index	Configuration/Level		NIST	GRASP				FAC1	FAC2	MCHF
				n≤3	n≤4	n≤5	n≤6			
1	$3s^2 3p$	$^2P^o_{1/2}$	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
2	$3s^2 3p$	$^2P^o_{3/2}$	0.06875	0.06813	0.06799	0.06803	0.06803	0.06775	0.06781	0.06652
3	$3s3p^2$	$^4P_{1/2}$	1.46175	1.44049	1.44131	1.44240	1.44287	1.44622	1.44797	1.44781
4	$3s3p^2$	$^4P_{3/2}$	1.48771	1.46615	1.46689	1.46800	1.46847	1.47171	1.47348	1.47324
5	$3s3p^2$	$^4P_{5/2}$	1.52463	1.50302	1.50361	1.50472	1.50519	1.50823	1.51000	1.50870
6	$3s3p^2$	$^2D_{3/2}$	1.93237	1.93826	1.93670	1.93568	1.93604	1.93510	1.93613	1.92770
7	$3s3p^2$	$^2D_{5/2}$	1.93743	1.94327	1.94166	1.94064	1.94100	1.93997	1.94100	1.93237
8	$3s3p^2$	$^2S_{1/2}$	2.40990	2.45532	2.45313	2.45107	2.45146	2.44592	2.44402	2.41723
9	$3s3p^2$	$^2P_{1/2}$	2.56113	2.61311	2.61457	2.61268	2.61318	2.60584	2.60515	2.58795
10	$3s3p^2$	$^2P_{3/2}$	2.59912	2.65121	2.65282	2.65096	2.65146	2.64394	2.64336	2.62591
11	$3s^2 3d$	$^2D_{3/2}$	3.14674	3.22157	3.20450	3.19742	3.19736	3.18670	3.18453	
12	$3s^2 3d$	$^2D_{5/2}$	3.15170	3.22581	3.20878	3.20174	3.20167	3.19080	3.18865	
13	$3p^3$	$^2D^o_{3/2}$	3.76715	3.76133	3.76590	3.76688	3.76910	3.76499	3.76612	3.77601
14	$3p^3$	$^2D^o_{5/2}$	3.77597	3.76965	3.77416	3.77516	3.77737	3.77324	3.77440	3.78481
15	$3p^3$	$^4S^o_{3/2}$	3.86116	3.87430	3.88360	3.88542	3.88867	3.88185	3.88365	3.87755
16	$3s3p(^3P)3d$	$^4F^o_{3/2}$		4.23096	4.23422	4.23471	4.23503	4.22902	4.22242	4.22677
17	$3s3p(^3P)3d$	$^4F^o_{5/2}$	4.24568	4.24533	4.24850	4.24900	4.24930	4.24323	4.23662	4.24080
18	$3p^3$	$^2P^o_{1/2}$	4.21135	4.24673	4.25390	4.25388	4.25703	4.24483	4.24205	4.22627
19	$3p^3$	$^2P^o_{3/2}$	4.21651	4.25128	4.25801	4.25794	4.26101	4.24920	4.24638	4.21328
20	$3s3p(^3P)3d$	$^4F^o_{7/2}$	4.26659	4.26602	4.26910	4.26960	4.26990	4.26371	4.25709	4.26121

Table 1. Energies (Ryd) for the lowest 40 levels of Ti X. – Part 2

Index	Configuration/Level		NIST	GRASP				FAC1	FAC2	MCHF
				$n \leq 3$	$n \leq 4$	$n \leq 5$	$n \leq 6$			
21	$3s3p(^3P)3d$	$^4F^o_{9/2}$	4.29466	4.29389	4.29688	4.29741	4.29770	4.29136	4.28473	
22	$3s3p(^3P)3d$	$^4P^o_{5/2}$	4.56977	4.58711	4.58514	4.58325	4.58357	4.57605	4.57617	4.57983
23	$3s3p(^3P)3d$	$^4P^o_{3/2}$	4.58313	4.60234	4.60023	4.59837	4.59868	4.59119	4.59057	4.59266
24	$3s3p(^3P)3d$	$^4P^o_{1/2}$	4.59427	4.61600	4.61382	4.61202	4.61233	4.60481	4.60319	4.62599
25	$3s3p(^3P)3d$	$^4D^o_{1/2}$	4.61875	4.63873	4.63650	4.63477	4.63507	4.62744	4.62590	4.60410
26	$3s3p(^3P)3d$	$^4D^o_{3/2}$	4.62461	4.64701	4.64472	4.64307	4.64338	4.63564	4.63305	4.63337
27	$3s3p(^3P)3d$	$^4D^o_{5/2}$	4.62795	4.65241	4.65007	4.64849	4.64879	4.64094	4.63748	4.63842
28	$3s3p(^3P)3d$	$^4D^o_{7/2}$	4.62755	4.65370	4.65128	4.64977	4.65007	4.64209	4.63777	4.63994
29	$3s3p(^3P)3d$	$^2D^o_{3/2}$	4.72979	4.77302	4.77836	4.77853	4.77962	4.76812	4.75674	4.74696
30	$3s3p(^3P)3d$	$^2D^o_{5/2}$	4.73051	4.77340	4.77857	4.77872	4.77978	4.76834	4.75705	4.74756
31	$3s3p(^3P)3d$	$^2F^o_{5/2}$	4.94969	5.02720	5.02800	5.02646	5.02665	5.01582	4.98724	4.96233
32	$3s3p(^3P)3d$	$^2F^o_{7/2}$	5.00420	5.08155	5.08226	5.08075	5.08093	5.06991	5.04120	5.01488
33	$3s3p(^3P)3d$	$^2P^o_{3/2}$	5.38048	5.49384	5.48253	5.47596	5.47706	5.46107	5.44869	5.40594
34	$3s3p(^3P)3d$	$^2P^o_{1/2}$	5.40519	5.51960	5.50549	5.49813	5.49911	5.48354	5.47274	5.42944
35	$3s3p(^1P)3d$	$^2F^o_{7/2}$	5.42225	5.55885	5.52829	5.51699	5.51441	5.50347	5.49264	5.45541
36	$3s3p(^1P)3d$	$^2F^o_{5/2}$	5.43543	5.57223	5.54160	5.53031	5.52771	5.51681	5.50590	5.46729
37	$3s3p(^1P)3d$	$^2P^o_{1/2}$	5.58268	5.72223	5.70877	5.70324	5.70363	5.69039	5.65982	5.62072
38	$3s3p(^1P)3d$	$^2P^o_{3/2}$	5.58836	5.72839	5.71301	5.70679	5.70711	5.69396	5.66493	5.62813
39	$3s3p(^1P)3d$	$^2D^o_{3/2}$	5.61581	5.74640	5.74873	5.74677	5.74787	5.73028	5.69535	5.65955
40	$3s3p(^1P)3d$	$^2D^o_{5/2}$	5.62423	5.75402	5.75697	5.75507	5.75616	5.73843	5.70343	5.66914

NIST: http://physics.nist.gov/PhysRefData
GRASP: present calculations from the GRASP code
FAC1: present calculations from the FAC code with **1304** levels
FAC2: present calculations from the FAC code with **12139** levels
MCHF: Froese−Fischer et al, ADNDT **92** (2006) 607
CIV3: Singh et al, ADNDT **96** (2010) 759

CI is not very important for Ti X

Table 2. Comparison of lifetimes (τ, ps) for the lowest 40 levels of Ti X. $a{\pm}b \equiv a{\times}10^{\pm b}$.

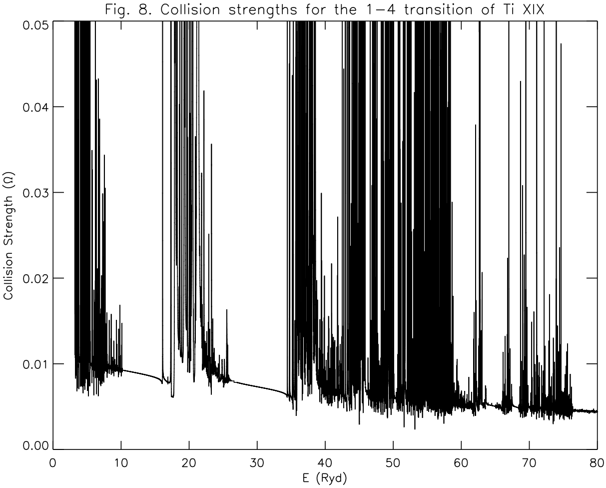
Index	Configuration/Level		GRASP n≤3	GRASP n≤4	GRASP n≤5	GRASP n≤6	FAC1	FAC2	CIV3	MCHF	MBPT	Experimental *a*	Experimental *b*
1	$3s^23p$	$^2P^o_{1/2}$											
2	$3s^23p$	$^2P^o_{3/2}$	2.663+11	2.679+11	2.675+11	2.675+11	2.708+11	2.701+11					
3	$3s3p^2$	$^4P_{1/2}$	1.966+05	1.890+05	1.857+05	1.855+05	1.817+05	1.791+05		1.680+05	2.12+05		
4	$3s3p^2$	$^4P_{3/2}$	9.567+05	9.435+05	9.273+05	9.265+05	9.151+05	8.992+05		8.895+05	1.09+05		
5	$3s3p^2$	$^4P_{5/2}$	3.171+05	3.043+05	2.973+05	2.973+05	2.941+05	2.891+05		2.840+05	3.75+05		
6	$3s3p^2$	$^2D_{3/2}$	8.329+02	8.070+02	8.004+02	8.019+02	8.085+02	7.965+02	9.399+02$\star$	7.624+02	9.21+02	[8.20±0.30]+02	[8.90±0.40,9.20±0.50,8.40±0.30,8.50±0.60]+02
7	$3s3p^2$	$^2D_{5/2}$	9.398+02	9.094+02	9.020+02	9.037+02	9.116+02	8.985+02	1.025+03	8.579+02	1.05+03	[8.70±0.40]+02	[9.70±0.30,9.80±0.50,8.50±0.60,9.50±0.50]+02
8	$3s3p^2$	$^2S_{1/2}$	9.925+01	9.674+01	9.630+01	9.627+01	9.676+01	9.676+01	1.138+02	1.012+02	1.08+02	[9.90±0.60]+01	[1.15±0.80,9.40±0.50,1.20±0.50,1.09±0.10]+01
9	$3s3p^2$	$^2P_{1/2}$	3.731+01	3.728+01	3.721+01	3.719+01	3.772+01	3.766+01	3.785+01	3.809+01	4.22+01	[6.00±0.15]+01	[6.00±0.40,6.60±0.50,6.70±0.30,4.30±0.50]+01
10	$3s3p^2$	$^2P_{3/2}$	3.550+01	3.556+01	3.550+01	3.548+01	3.596+01	3.593+01	3.748+01	3.652+01	4.02+01	[5.00±0.15]+01	[5.60±0.80,4.60±0.50,5.00±0.30,3.40±0.50]+01
11	$3s^23d$	$^2D_{3/2}$	3.048+01	3.087+01	3.087+01	3.084+01	3.123+01	3.126+01	3.135+01		3.52+01	[3.80±0.60]+01	[5.00±0.20,4.90±0.30,5.20±0.20,3.70±0.50]+01
12	$3s^23d$	$^2D_{5/2}$	3.208+01	3.251+01	3.251+01	3.248+01	3.291+01	3.293+01	3.311+01		3.72+01	[3.60±0.50]+01	[5.90±0.20,5.80±0.30,5.90±0.20,4.40±0.60]+01
13	$3p^3$	$^2D^o_{3/2}$	4.557+02	4.806+02	4.818+02	4.840+02	4.644+02	4.661+02	4.492+02	4.349+02		[4.70±0.50]+02	
14	$3p^3$	$^2D^o_{5/2}$	4.566+02	4.813+02	4.826+02	4.847+02	4.652+02	4.673+02	4.486+02	4.380+02		[4.90±0.40]+02	
15	$3p^3$	$^4S^o_{3/2}$	4.067+01	4.136+01	4.132+01	4.125+01	4.092+01	4.124+01	4.283+01	4.146+01			
16	$3s3p(^3P)3d$	$^4F^o_{3/2}$	1.200+04	1.347+04	1.321+04	1.400+04	1.148+04	1.285+04		8.226+03	1.77+04		[1.60±0.15]+04^c
17	$3s3p(^3P)3d$	$^4F^o_{5/2}$	1.619+04	1.618+04	1.596+04	1.597+04	1.599+04	1.612+04		1.740+04	1.88+04		[1.30±0.15]+04^c
18	$3p^3$	$^2P^o_{1/2}$	9.564+01	9.716+01	9.728+01	9.715+01	9.655+01	9.683+01	1.030+02	9.750+01		[1.10±0.10]+02	
19	$3p^3$	$^2P^o_{3/2}$	9.714+01	9.886+01	9.903+01	9.887+01	9.828+01	9.852+01	1.057+02	1.039+02			
20	$3s3p(^3P)3d$	$^4F^o_{7/2}$	1.873+04	1.853+04	1.823+04	1.823+04	1.830+04	1.861+04		2.089+04	2.20+04		[1.85±0.20]+04^c
21	$3s3p(^3P)3d$	$^4F^o_{9/2}$	7.060+10	7.155+10	7.119+10	7.122+10	7.230+10	7.319+10					
22	$3s3p(^3P)3d$	$^4P^o_{5/2}$	4.974+01	5.038+01	5.036+01	5.037+01	5.097+01	5.017+01	5.718+01$\star$	5.004+01	5.45+01		
23	$3s3p(^3P)3d$	$^4P^o_{3/2}$	4.431+01	4.494+01	4.497+01	4.499+01	4.548+01	4.407+01	5.528+01$\star$	4.393+01	4.76+01		
24	$3s3p(^3P)3d$	$^4P^o_{1/2}$	3.992+01	4.051+01	4.065+01	4.066+01	4.100+01	3.857+01	5.426+01$\star$	4.127+01	4.18+01		
25	$3s3p(^3P)3d$	$^4D^o_{1/2}$	3.748+01	3.824+01	3.800+01	3.802+01	3.863+01	4.118+01	3.132+01$\star$	3.840+01	4.51+01		
26	$3s3p(^3P)3d$	$^4D^o_{3/2}$	3.500+01	3.572+01	3.560+01	3.562+01	3.609+01	3.712+01	3.153+01$\star$	3.707+01	4.05+01		
27	$3s3p(^3P)3d$	$^4D^o_{5/2}$	3.314+01	3.386+01	3.379+01	3.380+01	3.421+01	3.464+01	3.175+01	3.449+01	3.80+01		
28	$3s3p(^3P)3d$	$^4D^o_{7/2}$	3.157+01	3.228+01	3.223+01	3.225+01	3.262+01	3.272+01	3.226+01	3.256+01	3.59+01		
29	$3s3p(^3P)3d$	$^2D^o_{3/2}$	3.836+01	3.850+01	3.832+01	3.827+01	3.855+01	3.886+01	4.098+01	3.972+01		[5.00±0.30]+01	
30	$3s3p(^3P)3d$	$^2D^o_{5/2}$	3.860+01	3.873+01	3.855+01	3.850+01	3.877+01	3.912+01	4.109+01	4.003+01		[5.00±0.30]+01	
31	$3s3p(^3P)3d$	$^2F^o_{5/2}$	7.873+01	7.912+01	7.879+01	7.898+01	7.978+01	8.036+01	8.537+01	8.911+01		[8.20±0.60]+01	
32	$3s3p(^3P)3d$	$^2F^o_{7/2}$	7.605+01	7.656+01	7.626+01	7.648+01	7.726+01	7.774+01	8.157+01	8.682+01		[8.80±0.80]+01	
33	$3s3p(^3P)3d$	$^2P^o_{3/2}$	2.685+01	2.665+01	2.665+01	2.662+01	2.690+01	2.671+01	2.695+01	2.765+01		[3.40±1.00]+01	
34	$3s3p(^3P)3d$	$^2P^o_{1/2}$	2.813+01	2.796+01	2.798+01	2.795+01	2.825+01	2.806+01	2.706+01	2.902+01		[4.70±0.40]+01	
35	$3s3p(^1P)3d$	$^2F^o_{7/2}$	2.465+01	2.540+01	2.546+01	2.553+01	2.570+01	2.580+01	2.638+01	4.200+01	3.03+01	[3.70±0.30]+01	
36	$3s3p(^1P)3d$	$^2F^o_{5/2}$	2.438+01	2.515+01	2.522+01	2.529+01	2.547+01	2.556+01	2.616+01	4.210+01	3.00+01		
37	$3s3p(^1P)3d$	$^2P^o_{1/2}$	2.115+01	2.144+01	2.147+01	2.145+01	2.166+01	2.226+01	2.273+01	4.045+01	3.57+01	[4.90±1.30]+01	
38	$3s3p(^1P)3d$	$^2P^o_{3/2}$	2.124+01	2.202+01	2.209+01	2.208+01	2.224+01	2.286+01	2.311+01	4.267+01	5.56+01	[4.90±1.30]+01	
39	$3s3p(^1P)3d$	$^2D^o_{3/2}$	1.703+01	1.680+01	1.670+01	1.669+01	1.684+01	1.720+01	1.670+01	2.315+01	2.11+01	[2.70±0.70]+01	
40	$3s3p(^1P)3d$	$^2D^o_{5/2}$	1.675+01	1.679+01	1.671+01	1.670+01	1.683+01	1.716+01	1.699+01	2.279+01	1.94+01	[2.70±0.70]+01	

GRASP: present calculations from the GRASP code
CIV3: Singh et al, J. Phys. **B 43** (2010) 11505
MCHF: Froese−Fischer et al, ADNDT **92** (2006) 607
a: Pinnington et al , Z. Phys. **D 6** (1987) 241
c: Träbert et al, Phys. Scr. **48** (1993) 593

FAC1: present calculations from the FAC code with **1304** levels,
FAC2: calculations with **12139** levels
MBPT: Safronova et al, ADNDT **84** (2003) 1
b: Pinnington et al, Z. Phys. **D 17** (1990) 5, the first entry is for Free M-E Fit, the second for Constrained M-E Fit, the 3rd for VNET, and the 4th for ANDC

CIV3 lifetimes differ by up to 30%

Figure 8: Collision strengths for the $2s^2\ ^1S_0$ - $2s2p\ ^3P^o_2$ (1 – 4) transition of Ti XIX.



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Comparison of Υ values for transitions of Ti XIX. ($a\pm b \equiv a\times10^{\pm b}$). – Part 1

log T_e (K)		6.3			6.9			7.5		
i	*j*	DARC	FAC	ZS	DARC	FAC	ZS	DARC	FAC	ZS
1	2	3.550−3	1.693−3	1.7943−3	2.753−3	1.242−3	1.3135−3	1.240−3	6.553−4	6.9065−4
1	3	1.480−2	8.981−3	9.1369−3	1.326−2	8.517−3	8.6113−3	9.330−3	8.208−3	8.1786−3
1	4	2.041−2	8.224−3	8.7866−3	1.497−2	6.031−3	6.4270−3	6.568−3	3.180−3	3.3795−3
1	5	4.632−1	4.683−1	4.4407−1	5.414−1	5.558−1	5.4721−1	6.535−1	7.149−1	7.1229−1
1	6	2.607−4⋆	1.126−4	1.1731−4	2.678−4	8.760−5	8.9602−5	1.421−4	6.029−5	5.9662−5
1	7	5.441−4⋆	2.059−4	2.2788−4	5.371−4	1.369−4	1.5205−4	2.272−4	6.298−5	6.7922−5
1	8	1.003−3	5.224−4	4.8162−4	1.002−3	4.565−4	3.9854−4	6.263−4	4.007−4	3.2024−4
1	9	2.566−3	1.645−3	1.2143−3	2.660−3	1.826−3	1.3591−3	2.379−3	2.116−3	1.5968−3
1	10	1.537−3	6.803−4	5.7142−4	1.550−3	6.579−4	5.5271−4	1.102−3	6.290−4	5.3012−4
2	3	4.162−2⋆	1.801−2	1.9718−2	2.591−2	1.262−2	1.3622−2	1.118−2	6.353−3	6.8570−3
2	4	3.037−2⋆	1.380−2	1.3591−2	2.181−2	1.266−2	1.2352−2	1.490−2	1.192−2	1.1670−2
2	5	9.787−3	4.090−3	4.4369−3	8.125−3	2.829−3	3.0501−3	3.445−3	1.387−3	1.4701−3
2	6	2.106−3	1.079−3	1.1455−3	1.733−3	7.869−4	8.3760−4	7.891−4	4.119−4	4.3712−4
2	7	2.202−1	2.226−1	2.2221−1	2.571−1	2.651−1	2.7186−1	3.058−1	3.409−1	3.4776−1
2	8	5.388−3	2.428−3	2.5623−3	4.697−3	1.774−3	1.8632−3	2.101−3	9.312−4	9.7530−4
2	9	3.211−3	1.243−3	1.3231−3	2.916−3	9.069−4	9.6379−4	1.262−3	4.755−4	5.0398−4
2	10	5.646−4⋆	1.509−4	1.5738−4	4.844−4	1.072−4	1.1212−4	1.976−4	5.397−5	5.6180−5
3	4	1.550−1⋆	5.275−2	5.4204−2	9.090−2	4.364−2	4.4300−2	4.970−2	3.436−2	3.4175−2
3	5	3.741−2⋆	1.261−2	1.3622−2	2.691−2	8.896−3	9.5108−3	1.147−2	4.685−3	4.9728−3
3	6	2.262−1	2.282−1	2.2576−1	2.631−1	2.725−1	2.7867−1	3.112−1	3.505−1	3.5752−1
3	7	1.729−1	1.702−1	1.6936−1	1.991−1	2.011−1	2.0605−1	2.307−1	2.564−1	2.6172−1
3	8	2.864−1	2.836−1	2.8422−1	3.313−1	3.355−1	3.4509−1	3.849−1	4.288−1	4.3893−1
3	9	1.843−2	1.106−2	1.0630−2	1.805−2	1.082−2	1.0263−2	1.354−2	1.111−2	1.0400−2
3	10	2.511−3⋆	8.760−4	9.0560−4	2.237−3	7.512−4	7.8159−4	1.196−3	6.309−4	6.5237−4

DARC: Present calculations from the DARC code
FAC: Present calculations from the FAC code
ZS: Calculations of Zhang and Sampson (1992)

Comparison of Υ values for transitions of Ti XIX. ($a{\pm}b \equiv a{\times}10^{\pm b}$). – Part 2

log T_e (K)		6.3			6.9			7.5		
i	*j*	DARC	FAC	ZS	DARC	FAC	ZS	DARC	FAC	ZS
4	5	7.273−2⋆	2.146−2	2.3186−2	4.562−2	1.488−2	1.6003−2	1.844−2	7.398−3	7.8110−3
4	6	3.254−3⋆	8.655−4	9.4764−4	2.512−3	6.355−4	6.9369−4	1.052−3	3.358−4	3.6439−4
4	7	2.884−1	2.858−1	2.8227−1	3.333−1	3.407−1	3.4819−1	3.890−1	4.371−1	4.4551−1
4	8	7.430−1	7.522−1	7.5185−1	8.635−1	8.957−1	9.2247−1	1.014+0	1.150+0	1.1785+0
4	9	1.280−1	1.069−1	1.0317−1	1.393−1	1.202−1	1.1693−1	1.469−1	1.465−1	1.4356−1
4	10	6.400−3⋆	1.849−3	1.9425−3	5.279−3	1.328−3	1.3970−3	2.164−3	6.808−4	7.1247−4
5	6	9.319−3	6.618−3	7.2781−3	9.312−3	7.599−3	8.5691−3	8.387−3	9.108−3	1.0238−2
5	7	1.407−2	5.982−3	6.2057−3	1.208−2	5.274−3	5.4767−3	6.677−3	4.463−3	4.6440−3
5	8	1.270−1	1.116−1	1.1022−1	1.378−1	1.338−1	1.3584−1	1.462−1	1.687−1	1.7047−1
5	9	1.074+0	1.094+0	1.0744+0	1.237+0	1.328+0	1.3820+0	1.422+0	1.717+0	1.7953+0
5	10	3.799−1	3.868−1	3.9823−1	4.431−1	4.613−1	4.8814−1	5.268−1	5.939−1	6.2552−1
6	7	3.074−2	1.923−2	2.0870−2	2.496−2	1.359−2	1.4672−2	1.144−2	6.908−3	7.3534−3
6	8	2.441−2	1.730−2	1.7613−2	2.226−2	1.526−2	1.5500−2	1.620−2	1.347−2	1.3428−2
6	9	1.137−2	6.189−3	6.9335−3	9.442−3	4.247−3	4.7452−3	4.080−3	2.062−3	2.2528−3
6	10	3.660−3⋆	9.244−4	1.0278−3	3.010−3	6.115−4	6.7632−4	1.169−3	2.795−4	3.0982−4
7	8	8.444−2	5.652−2	5.9629−2	7.490−2	4.555−2	4.7635−2	4.542−2	3.381−2	3.4716−2
7	9	5.030−2	2.925−2	3.1902−2	4.358−2	2.095−2	2.2555−2	2.079−2	1.163−2	1.2341−2
7	10	1.383−2⋆	3.944−3		1.088−2	2.632−3		4.220−3	1.221−3	
8	9	1.013−1	6.637−2	7.0516−2	8.985−2	5.088−2	5.3039−2	5.028−2	3.388−2	3.3996−2
8	10	1.998−2	8.238−3	8.7911−3	1.620−2	6.240−3	6.4961−3	7.948−3	4.246−3	4.2710−3
9	10	3.972−2	2.929−2	2.8800−2	4.004−2	3.092−2	3.0604−2	3.769−2	3.457−2	3.4692−2

DARC: Present calculations from the DARC code
FAC: Present calculations from the FAC code
ZS: Calculations of Zhang and Sampson (1992)

FAC and ZS agree within ∼20% but DARC Υ are higher by up to a factor of 4

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Table 1b. Experimental and theoretical energy levels (in Ryd) for Ti XX and their lifetimes (τ, s). $a{\pm}b \equiv a{\times}10^{\pm b}$

Index	Configuration	Level	NIST	GRASP1	GRASP2	FAC	BPRM	τ (s)
1	$1s^22s$	$^2S_{1/2}$	0.00000	0.00000	0.00000	0.00000	0.00000	....
2	$1s^22p$	$^2P^o_{1/2}$	2.94814	2.95843	2.96034	2.97295	2.90900	6.708-10
3	$1s^22p$	$^2P^o_{3/2}$	3.51495	3.54785	3.52549	3.53597	3.55600	3.930-10
4	$1s^23s$	$^2S_{1/2}$	58.92244	58.94397	58.91143	58.91679	58.96600	7.279-13
5	$1s^23p$	$^2P^o_{1/2}$	59.73921	59.76326	59.73184	59.73847	59.75200	2.796-13
6	$1s^23p$	$^2P^o_{3/2}$	59.90706	59.93739	59.89893	59.90476	59.94280	2.863-13
7	$1s^23d$	$^2D_{3/2}$	60.21762	60.24985	60.20592	60.20768	60.24240	9.412-14
8	$1s^23d$	$^2D_{5/2}$	60.27047	60.30285	60.25860	60.26015	60.30120	9.490-14
9	$1s^24s$	$^2S_{1/2}$	79.16195	79.19708	79.15763	79.15313	78.84310	1.074-12
10	$1s^24p$	$^2P^o_{1/2}$	79.50504	79.53436	79.49532	79.48676	79.47540	4.732-13
11	$1s^24p$	$^2P^o_{3/2}$	70.57585	79.60766	79.56569	79.55669	79.55750	4.827-13
12	$1s^24d$	$^2D_{3/2}$	79.70671	79.73733	79.69318	79.69778	79.75280	2.183-13
13	$1s^24d$	$^2D_{5/2}$	79.72894	79.75972	79.71540	79.71973	79.77760	2.203-13
14	$1s^24f$	$^2F^o_{5/2}$	79.73587	79.76650	79.72211	79.71511	79.78310	4.509-13
15	$1s^24f$	$^2F^o_{7/2}$	79.74498	79.77762	79.73322	79.72622	79.79539	4.520-13
16	$1s^25s$	$^2S_{1/2}$	88.44913	88.48424	88.44234	88.43439	88.33580	1.695-12
17	$1s^25p$	$^2P^o_{1/2}$	88.62437	88.65482	88.61314	88.60387	88.63940	8.021-13
18	$1s^25p$	$^2P^o_{3/2}$	88.66054	88.69228	88.64912	88.63941	88.68069	8.169-13
19	$1s^25d$	$^2D_{3/2}$	88.72752	88.75816	88.71390	88.71542	88.76910	4.190-13
20	$1s^25d$	$^2D_{5/2}$	88.73891	88.76962	88.72527	88.72663	88.78180	4.232-13
21	$1s^25f$	$^2F^o_{5/2}$		88.77345	88.72904	88.72179	88.78500	8.712-13
22	$1s^25f$	$^2F^o_{7/2}$		88.77914	88.73473	88.72747	88.79120	8.736-13
23	$1s^25g$	$^2G_{7/2}$		88.77921	88.73480	88.72717	88.79090	1.465-12
24	$1s^25g$	$^2G_{9/2}$		88.78262	88.73821	88.73058	88.79460	1.467-12

NIST: http://www.nist.gov/pml/data/asd.cfm
GRASP1: Present results from the GRASP code *without* the Breit and QED effects
GRASP2: Present results from the GRASP code *with* the Breit and QED effects
FAC: Present results from the FAC code
BPRM: Nahar (2002)

Relativistic effects are not too important for Ti XX

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b. Comparison of radiative rates (A- values, s^{-1}) for some transitions of Ti XX. ($a{\pm}b \equiv a{\times}10^{\pm b}$).

i	*j*	A (GRASP)	A (FAC)	A (BPRM)	*i*	*j*	A (GRASP)	A (FAC)	A (BPRM)
1	2	1.4907+09	1.4990+09	1.4030+09	7	21	6.8262+11	6.8360+11	6.8420+11
1	3	2.5447+09	2.5500+09	2.5930+09	8	14	1.4706+11	1.4680+11	1.4750+11
1	5	3.5749+12	3.6310+12	3.6770+12	8	15	2.2079+12	2.2080+12	2.2150+12
1	6	3.4911+12	3.5540+12	3.5830+12	8	21	4.8404+10	4.8380+10	4.8480+10
1	10	1.5800+12	1.6500+12	1.3880+12	8	22	7.2830+11	7.2980+11	7.2960+11
1	11	1.5542+12	1.6270+12	1.3630+12	9	10	4.5265+07		2.9160+08$\star$
1	17	8.1676+11	9.1200+11	7.4220+11	9	11	8.0239+07		4.2270+08$\star$
1	18	8.0541+11	9.0220+11	7.3070+11	9	17	1.0936+11	1.1230+11	1.0170+11
2	7	8.8657+12	8.9040+12	8.8800+12	9	18	1.0607+11	1.0900+11	9.7840+10
2	12	2.8917+12	2.9410+12	2.8480+12	10	12	7.4663+06		2.0560+07$\star$
2	19	1.3340+12	1.4250+12	1.3250+12	10	19	1.9329+11	1.9710+11	1.9160+11
3	8	1.0533+13	1.0590+13	1.0540+13	11	13	3.8843+06		1.2340+07$\star$
3	13	3.4188+12	3.4810+12	3.3630+12	11	20	2.3314+11	2.3760+11	2.3150+11
3	20	1.5731+12	1.6780+12	1.5610+12	12	14	1.6307+04		1.8750+04
4	5	1.9328+08		1.6820+08	12	18	3.3472+09	3.4160+09	1.8630+11$\star$
4	6	3.3925+08		3.2530+08	12	21	3.8650+11	3.8550+11	3.8590+11
4	10	4.6591+11	4.7240+11	4.3630+11	13	14	1.4472+01		7.7210+00$\star$
4	11	4.5300+11	4.5970+11	4.2210+11	13	15	4.0837+03		3.9990+03
4	17	2.5631+11	2.7290+11	2.4470+11	13	21	2.7602+10	2.7510+10	2.7560+10
4	18	2.5088+11	2.6730+11	2.3870+11	13	22	4.1369+11	4.1270+11	4.1310+11
5	7	2.4095+07		2.6690+07	14	23	6.5745+11	6.5750+11	6.5660+11
5	12	9.3273+11	9.3960+11	9.2720+11	16	17	1.4704+07		8.2050+07
5	19	4.5946+11	4.7490+11	4.6000+11	16	18	2.6185+07		1.2070+08$\star$
6	8	1.2640+07		1.2510+07	17	19	2.6949+06		5.7560+06
6	13	1.1192+12	1.1280+12	1.1130+12	18	20	1.3974+06		3.2770+06
6	20	5.4834+11	5.6610+11	5.4910+11	19	21	8.3660+03		9.5790+03
7	11	6.2633+09	6.3600+09	1.7550+12$\star$	20	21	9.2274+00		5.0810+00
7	14	2.0665+12	2.0650+12	2.0740+12	20	22	2.1866+03		2.1250+03
7	18	2.6896+09	2.9250+09	1.2150+06$\star$	21	23	2.7721+02		3.0120+02

GRASP: Present results from the GRASP code
FAC: Present results from the FAC code
RMBP: Nahar, A&A **389** (2002) 716

BPRM A-values differ by up to 3 orders of magnitude

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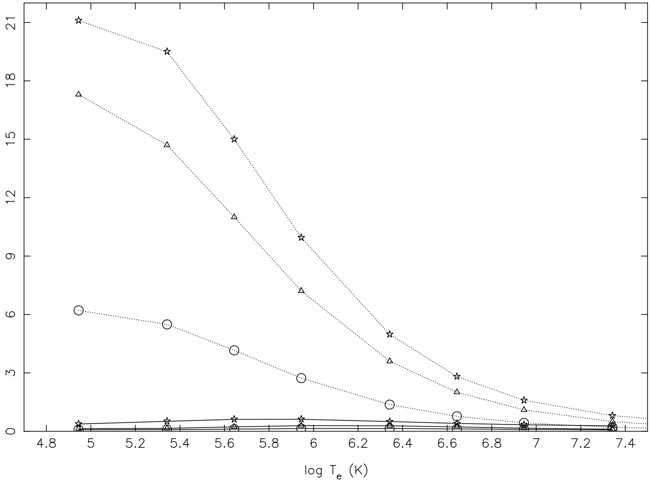
Figure 10. Comparison of effective collision strengths for the 13–14, 14–16, and 24–26 transitions of Ti XXI.



Continuous Curves: DARC
Broken Curves: BPRM

circles: 13–14
(1s3d $^3D_1$ - 1s3d $^3D_2$)
triangles: 14–16
(1s3d $^3D_2$ - 1s3d $^3D_3$)
stars: 24–26
(1s4d $^3D_2$ - 1s4d $^3D_3$)

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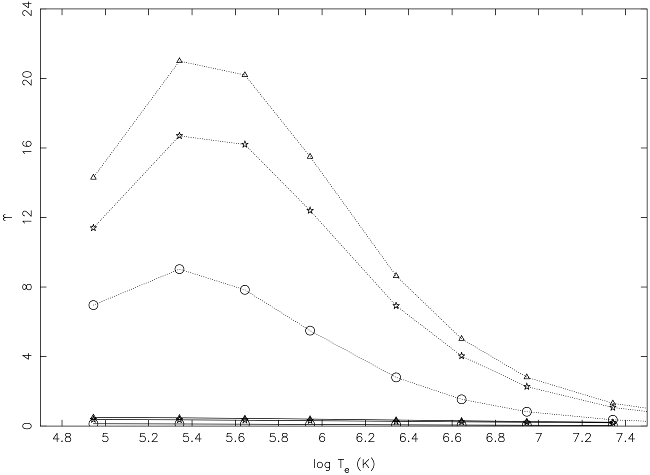
Figure 11. Comparison of effective collision strengths for the 46–48, 47–48, and 48–49 transitions of Ti XXI.



Continuous Curves: DARC
Broken Curves: BPRM

circles: 46–48
(1s5g $^3G_3$ - 1s5g $^3G_5$)
triangles: 47–48
(1s5g $^3G_4$ - 1s5g $^3G_5$)
stars: 48–49
(1s5g $^3G_5$ - 1s5g $^1G_4$)

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Conclusions

- Ω and Υ need to be calculated for Ti VI and Ti X
- Scope remains for improvement in the collisional data for Ti XIX, Ti XX and Ti XXI