Developments in beam models
Motion Stark Effect

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Contents

Contents

The Motion Stark Effect (MSE)

In fusion devices, like tokamaks, Neutral Beam Injectors (NBI) insert high-energy neutral atoms inside the magnetic confined plasma.
As the atoms are neutrals, they do not react as a hole system to these magnetic fields, being able to penetrate deeply into the plasma until they are ionised.
Internally, the neutrals can fell simultaneous electric and magnetic fields, which disturb their electronic structure.
The atom is moving rapidly under an intense magnetic field, what causes a Lorentz electric field.
The atom is under the influence of simultaneous electric and magnetic fields.

MSE spectrum diagnostic

MSE spectroscopy to determine magnetic and electric fields in ASDEX-Upgrade Tokamak.

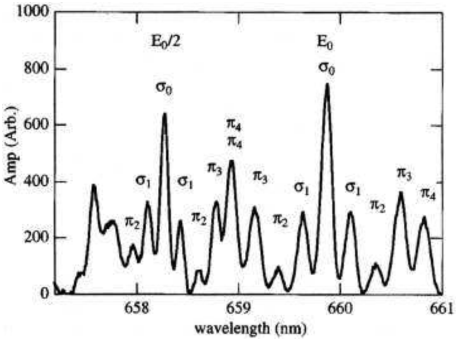

Figure: MSE spectrum of D_α line

Diagnostic setup overview

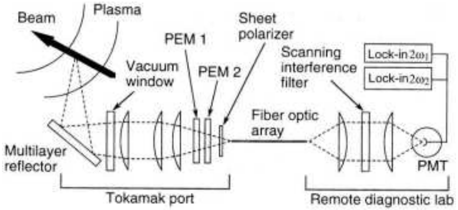

Figure: Schematic overview of MSE diagnostic in ASDEX-Upgrade Tokamak

MSE lines of sight

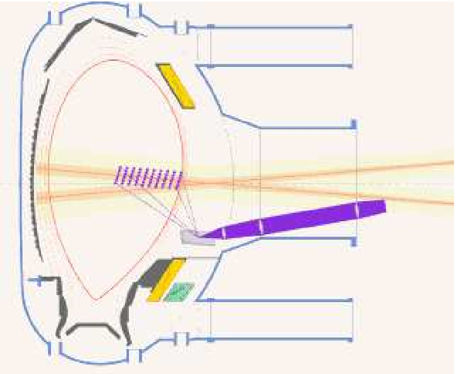

Figure: Poloidal overview of MSE sight lines

MSE lines of sight

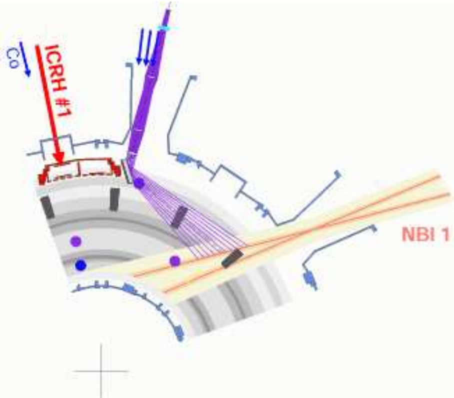

Figure: Toroidal overview of MSE sight lines

1914. № 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 43.

1. ***Beobachtungen über den Effekt des elektrischen Feldes auf Spektrallinien. I. Quereffekt;***[1])
von J. Stark.

PHYSICAL REVIEW A **88**, 022509 (2013)

Stark effect in neutral hydrogen by direct integration of the Hamiltonian in parabolic coordinates

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(Received 18 June 2013; published 19 August 2013)

We present a theoretical study to determine the energies, widths, and wave functions of the neutral hydrogen atom under a constant electric field by the direct integration of the Hamiltonian in parabolic coordinates. We work in terms of the complex coordinate rotation to distinguish the resonances from the continuum sea, and the wave functions are expanded in a basis set of Laguerre-mesh polynomials. We obtain *ab initio* results for the first five atomic shells of neutral hydrogen for a field intensity up to 10^9 V/m.

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Contents

The Stark hydrogen atom

$$x = \sqrt{\xi\eta}\cos\phi \quad , \quad y = \sqrt{\xi\eta}\sin\phi$$
$$z = \frac{\xi-\eta}{2} \quad , \quad r = \frac{\xi+\eta}{2}$$

RHA Rydberg hydrogen atom: Solution to the Schrödinger equation for the unperturbed hydrogen atom. Usual wave functions in spherical coordinates and labeled by quantum numbers n, l and m.

SHA Stark hydrogen atom: Solution to the Schrödinger equation for the hydrogen atom under a constant electric field, which can tend to zero. Wave functions described in parabolic coordinates and labeled by quantum numbers n, k and m.

Bethe, H. A. and Salpeter, E. E., 1957, Quantum Mechanics of One- and Two-Electron Systems, New York: Academic Press

Potential

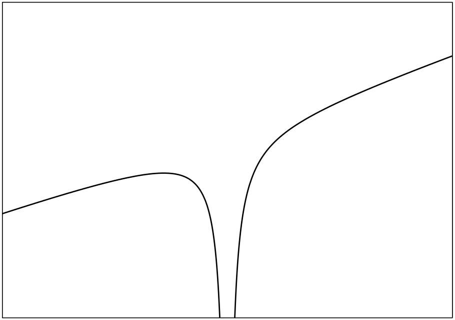


Figure: Diagram showing the behavior of a root in the Hydrogen atom under a constant electric field

Potential

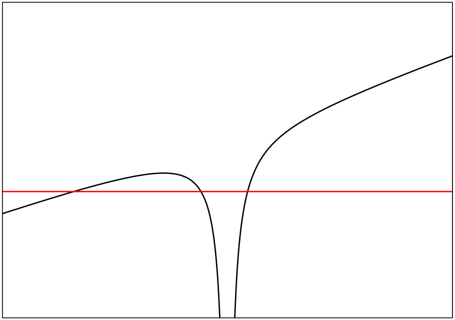
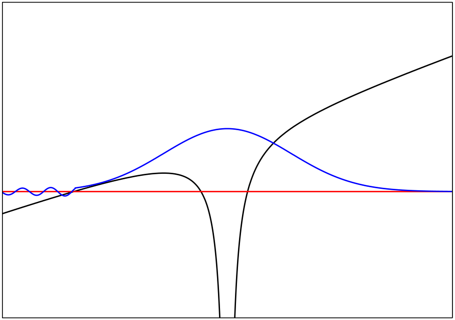


Figure: Diagram showing the behavior of a root in the Hydrogen atom under a constant electric field

Potential



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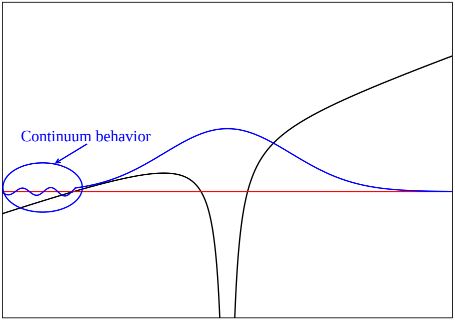

Figure: Diagram showing the behavior of a root in the Hydrogen atom under a constant electric field

Potential

Hydrogen atom under a constant electric field must be determined by any exact method beyond perturbation theory.

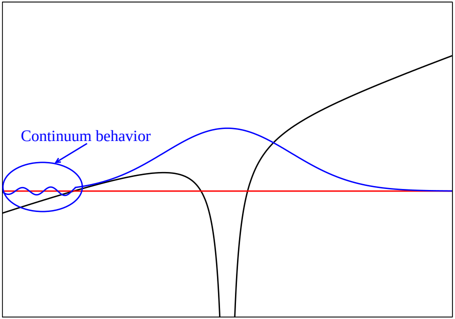

Figure: Diagram showing the behavior of a root in the Hydrogen atom under a constant electric field

SHA Hamiltonian. The complex coordinate method.

$$
\begin{aligned}
H_0 &= -\frac{1}{2\mu}\nabla^2 - \frac{1}{r} + F\,r\,\cos\theta \\
&= \frac{2}{\xi+\eta}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial}{\partial\xi}\right) - \frac{2}{\xi+\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial}{\partial\eta}\right) - \frac{1}{2\xi\eta}\frac{\partial^2}{\partial\varphi^2} \\
&\quad - \frac{2}{\xi+\eta} + \frac{1}{2}F\,(\xi - \eta)
\end{aligned}
$$

SHA Hamiltonian. The complex coordinate method.

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\end{aligned}
$$

Complex coordinate rotation.

$$
r' = r\,\mathrm{e}^{i\vartheta}
$$

SHA Hamiltonian. The complex coordinate method.

$$
\begin{aligned}
H_0(\vartheta) &= -\frac{\mathrm{e}^{-2i\vartheta}}{2\mu}\nabla^2 - \frac{\mathrm{e}^{-i\vartheta}}{r} + \mathrm{e}^{i\vartheta} F\, r \cos\theta \\
&= \frac{2\,\mathrm{e}^{-2i\vartheta}}{\xi+\eta}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial}{\partial\xi}\right) - \frac{2\,\mathrm{e}^{-2i\vartheta}}{\xi+\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial}{\partial\eta}\right) - \frac{\mathrm{e}^{-2i\vartheta}}{2\xi\eta}\frac{\partial^2}{\partial\varphi^2} \\
&\quad - \frac{2\,\mathrm{e}^{-i\vartheta}}{\xi+\eta} + \frac{\mathrm{e}^{i\vartheta}}{2} F\,(\xi - \eta)
\end{aligned}
$$

Complex coordinate rotation.

$$
r' = r\,\mathrm{e}^{i\vartheta}
$$

Variational method: basis set

$$\Psi(\xi,\eta,\varphi) \;=\; \frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{im\varphi}\,(\xi\eta)^{\frac{|m|}{2}}\,\mathrm{e}^{-\frac{\xi+\eta}{2}}\,\sum_{k=1}^{N}\sum_{l=1}^{N} c_{klm}\,\Lambda_{Nk}(\xi)\,\Lambda_{Nl}(\eta)$$

Lagrange-Laguerre-mesh polynomials:

$$\Lambda_{Ni}(x) \;=\; (-1)^i\sqrt{x_i}\,\frac{\mathrm{L}_N(x)}{x - x_i}$$

x_i: zeros of the Laguerre polynomial $\mathrm{L}_N(x)$.

Secular equation

$$S_{klk'l'm} = \left\langle (\xi\eta)^{\frac{|m|}{2}} \, \mathrm{e}^{-\frac{\xi+\eta}{2}} \, \Lambda_{Nk} \, \Lambda_{Nl} \right| \left. (\xi\eta)^{\frac{|m|}{2}} \, \mathrm{e}^{-\frac{\xi+\eta}{2}} \, \Lambda_{Nk} \, \Lambda_{Nl} \right\rangle$$

$$H_{klk'l'm}(\vartheta) = \left\langle (\xi\eta)^{\frac{|m|}{2}} \, \mathrm{e}^{-\frac{\xi+\eta}{2}} \, \Lambda_{Nk} \, \Lambda_{Nl} \right| \hat{H}(\vartheta) \left| (\xi\eta)^{\frac{|m|}{2}} \, \mathrm{e}^{-\frac{\xi+\eta}{2}} \, \Lambda_{Nk} \, \Lambda_{Nl} \right\rangle$$

Secular equation

$$(\mathbf{H} - \mathbf{E}\,\mathbf{S})\,\mathbf{C} = 0$$

For further detail:
L. F. Menchero and H. P. Summers, Max Planck Institute for Plasma Physics, Report No. IPP-10/49, 2013, internal report, http://edoc.mpg.de/display.epl?mode=doc&id=656145.

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Contents

Found eigenvalues

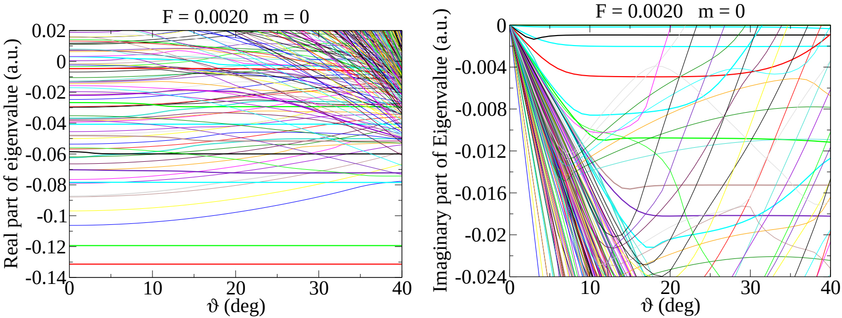

Figure: Obtained eigenvalues for a basis set $N = 30$ for $m = 0$ and a field intensity $F = 0.0020\,\mathrm{a.u}$. Marked some found resonances.

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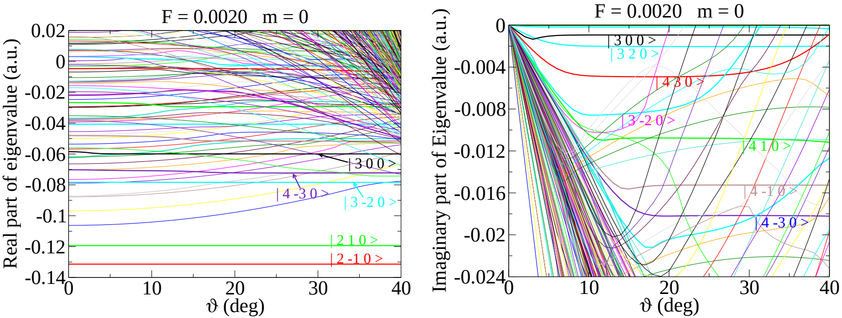
Found eigenvalues



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Energies

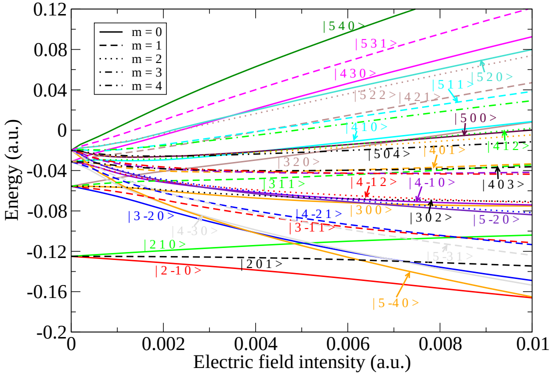

Figure: State energies of the H atom versus electric field intensity for $m = 0 - 4$.

Widths

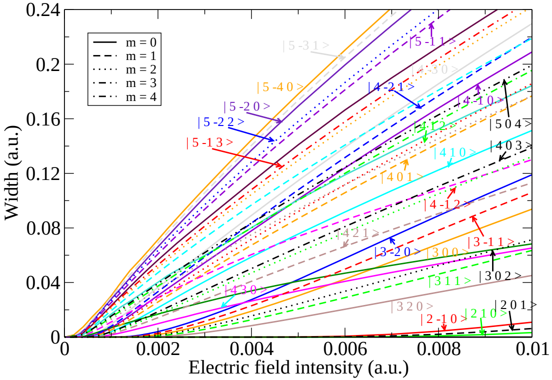

Figure: State widths of the H atom versus electric field intensity for $m = 0 - 4$.

Wave functions

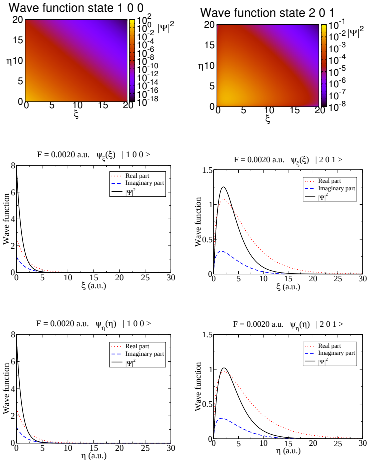

Figure: Wave function of the Stark states $|1\,0\,0\rangle$ and $|2\,0\,1\rangle$ for a field intensity of $F = 0.0020\,\mathrm{a.u.}$.

Wave functions

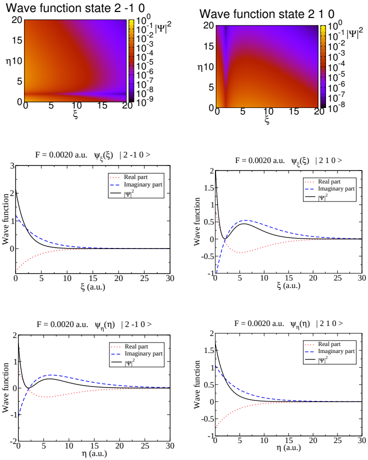

Figure: Wave function of the Stark states $|2\,-1\,0\rangle$ and $|2\,1\,0\rangle$ for a field intensity of $F = 0.0020\,\mathrm{a.u.}$.

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Einstein transition coefficients

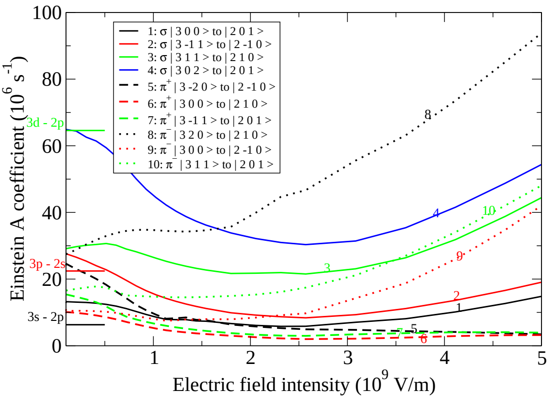

Figure: Einstein spontaneous emission coefficients of neutral hydrogen versus the electric field intensity. Marked the values for the Rydberg Hydrogen Atom for zero field intensity.

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Balmer D_α line splitting

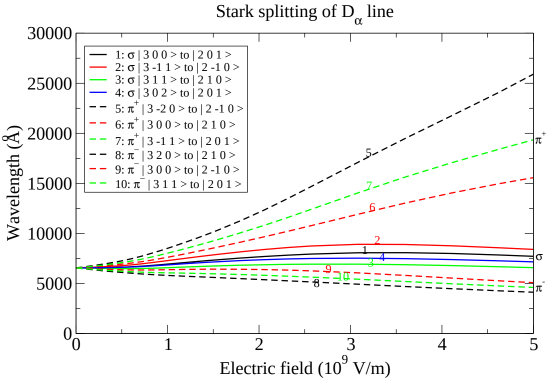

Figure: Stark splitting of the D_α line of deuterium versus the electric field intensity.

Balmer D_α line splitting

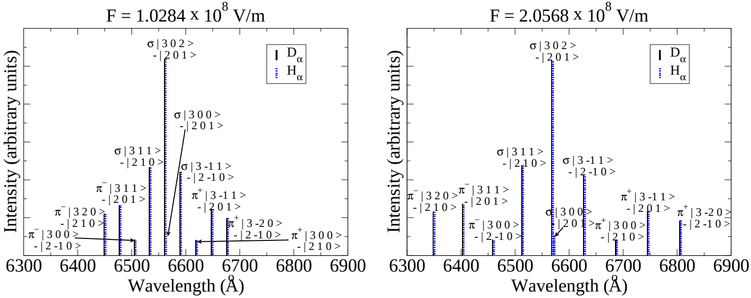

Figure: Emission profile of the Balmer D_α and H_α lines for two different field intensities in corona equilibrium.

Contents

Conclusions

- A method to solve the hydrogen atom under a constant electric field has been developed beyond perturbation theory.
- Stark energies, widths and wave functions *nkm* have been determined up to $n = 5$.
- Wave functions can be used to get any physical observable.
- Further effects (fine structure, static magnetic field) can be added as perturbations of the Stark wave functions.
- A background has been prepared to develop a collision-radiative model for hydrogen atom under a constant electric field.
- Results are collected in adf50 format.

Further work

- Use the obtained wave functions to calculate directional cross sections of collision with SHA: electron impact, ion impact, charge exchange.
- Include these cross sections and Einstein coefficients in the collision-radiative model for hydrogen atom under constant symultaneous electric and magnetic field.
- Collect all in a second version of ADAS305.

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