Chapter 5

Comments on the effects of structure and flow

In chapter 2 escape probability and absorption factor techniques were introduced and applied to spectral data from the SOHO-SUMER spectrometer in both a diagnostic and modelling capacity. These analyses were effective in extracting optical depths of spectral lines of C II and C III and led to the classification of lines of these two ions based on the influence of opacity on both atomic population structures and emergent intensities. Also the techniques, when coupled with simple stratified atmosphere models were successful in describing observed C II and C III branching ratios of lines arising from common upper levels. However, inadequacies were found in the escape probability/absorption factor methods, such as their inability to extract optical depths from C III branching ratios in the vicinity of the limb and the failure to accurately predict emergent intensities. The relative simplicity of these methods, however, make them desirable and have prompted this study into their validity.

So far the influence of the spatial dependence of the source function due to opacity has been analysed as well as its frequency dependence through spectral line blending. These studies have led to the identification of an optical depth regime within which the line-of-sight averaged escape probability, $\bar{g}^{(i)}\{\tau_0\}$, is valid. However, some assumptions underpinning these expressions remain untested. Specifically the neglect of the spatial dependence of the source function due to the variation of (T_e, N_e) remains unjustified as do the effects of plasma flow and partial frequency redistribution. The latter two, like line blending lead to a dependence of the source function on frequency. In addition, the roles played by solar atmospheric structure and instrumental effects in interpreting and modelling spectral emission are yet to be addressed.

In this chapter the influence of atmospheric structure and plasma flow upon the escape probability approach are considered. The remaining issues listed above will be addressed in chapter 6.

5.1 Non-stratified models

The atmosphere models examined thus far have been simple but the only structural restriction on them, as far as the escape probability and absorption factor expressions are concerned, is that they are static and stratified. Therefore it is necessary to find a stratified model that best captures the radiative characteristics of the real solar atmosphere. For example, the exponential model described in chapter 2 is more effective than the VAL model in describing cross-limb flux ratio variations despite its empirical nature. This is because it takes account of the extention of the TR into the corona due to spicule-like structures.

However, the solar atmosphere is not stratified, nor is it static. A question therefore presents itself as to how structure and flow affect absorption characteristics in a plasma and the quantities derived thus far.

Absorption at a point is dependent upon the radiation field at that point. This radiation field is determined by the number of emitters in the plasma and the probability that photons emitted from them will reach the point in question. This probability is dependent upon the number of absorbers along the line-of-sight from the emitter to the absorber. Consider two plasmas of the same total number of emitters and absorbers, one stratified and one non-stratified. Absorption will be greater in the stratified plasma than the non-stratified one since in the latter, the particles will appear more overlapped along each line-of-sight. This may be shown mathematically as follows: Consider the absorption factor at layer centre. If the layer extends from

$x = -D/2$ to $x = D/2$ then this is given by

$$\Lambda(\tau_0, 0) = 1 - \frac{N_l(0)}{N_u(0)} \frac{\omega_u}{\omega_l} \frac{c^2}{2\nu_0^2} \int I_\nu \phi_\nu d\nu \tag{5.1}$$

where

$$I_\nu = \frac{1}{4\pi} \int\int_V\int \frac{j_\nu(\mathbf{r})}{r^2} e^{-\tau_\nu(\mathbf{r})} dV \tag{5.2}$$

If it is written that

$$\tau_\nu(\mathbf{r}) = \alpha_\nu \int_{\mathbf{r}\to\mathbf{0}} N_l(\mathbf{r}') dr' \tag{5.3}$$

and

$$j_\nu(\mathbf{r}) = \beta_\nu N_u(\mathbf{r}) \tag{5.4}$$

where

$$\alpha_\nu = \frac{1}{c} h\nu B_{l\to u} \phi(\nu) \tag{5.5}$$

and

$$\beta_\nu = \frac{1}{4\pi} A_{u\to l} \phi(\nu) \tag{5.6}$$

then for a semi-infinite slab

$$I_\nu = \frac{\beta_\nu}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \int_0^{R/\cos\theta} N_u(\mathbf{r}) exp\left\{-\alpha_\nu \int_0^r N_l(\mathbf{r}')dr'\right\} |\sin\theta| dr d\theta d\phi \tag{5.7}$$

It is useful to define $\bar{N}_l(\theta, \phi)$ and $\bar{N}_u(\theta, \phi)$ as

$$\bar{N}_l(\theta, \phi) = \frac{\int_0^{R/\cos\theta} N_l(\mathbf{r})dr}{R/\cos\theta} \tag{5.8}$$

$$\bar{N}_u(\theta, \phi) = \frac{\int_0^{R/\cos\theta} N_u(\mathbf{r})dr}{R/\cos\theta} \tag{5.9}$$

and then to define s as

$$s = \frac{\int_0^r N_l(\mathbf{r}')dr'}{\bar{N}_l(\theta, \phi)} \tag{5.10}$$

$$\Rightarrow \frac{ds}{dr} = \frac{N_l(\mathbf{r})}{\bar{N}_l(\theta, \phi)} \tag{5.11}$$

$$\Rightarrow I_\nu = \frac{\beta_\nu}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \int_0^{R/\cos\theta} \frac{N_u(\mathbf{r})}{N_l(\mathbf{r})} \bar{N}_l(\theta, \phi) e^{-\alpha_\nu \bar{N}_l(\theta,\phi) s} |\sin\theta| ds d\theta d\phi \tag{5.12}$$

Now consider the regime where the source function is approximately constant. Then

$$\frac{N_u(\mathbf{r})}{N_l(\mathbf{r})} = \frac{\bar{N}_u(\theta,\phi)}{\bar{N}_l(\theta,\phi)} \tag{5.13}$$

and so

$$\begin{aligned}
I_\nu &= \frac{\beta_\nu}{4\pi}\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2}\int_0^{R/\cos\theta} \bar{N}_u(\theta,\phi)e^{-\alpha_\nu \bar{N}_l(\theta,\phi)s}|\sin\theta|dsd\theta d\phi \\
&= \frac{\beta_\nu}{4\pi}\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} \frac{\bar{N}_u(\theta,\phi)}{\bar{N}_l(\theta,\phi)}\frac{1}{\alpha_\nu}\left[1-e^{-\alpha_\nu \bar{N}_l(\theta,\phi)R/\cos\theta}\right]|\sin\theta|d\theta d\phi \\
&= \frac{S_\nu}{4\pi}\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2}\left[1-e^{-\alpha_\nu \bar{N}_l(\theta,\phi)R/\cos\theta}\right]|\sin\theta|d\theta d\phi
\end{aligned} \tag{5.14}$$

where S_ν is the source function. The question is now, how does structuring with respect to θ and ϕ influence the radiation field at a point? To answer this, consider the integral

$$\mathcal{I}_{\nu,\theta} = \int_{-\pi}^{\pi} e^{-\alpha_\nu \bar{N}_l(\theta,\phi)R/\cos\theta}d\phi \tag{5.15}$$

Intuitively, it is expected that structuring a plasma will lead to a reduction in the radiation field. This was discussed above. If this is so then introducing structure with respect to the angle ϕ in eq. 5.15 will lead to an increase in $\mathcal{I}_{\nu,\theta}$. This may be shown to be the case as follows: write

$$\mathcal{I}_{\nu,\theta} = \int_{-\pi}^{\pi} e^{-a_{\nu,\theta}\left(\bar{\bar{N}}_{l,\theta}+\varepsilon(\phi)\right)}d\phi \tag{5.16}$$

where

$$\begin{aligned}
a_{\nu,\theta} &= -\alpha_\nu\frac{R}{\cos\theta} &(5.17)\\
\bar{\bar{N}}_{l,\theta} &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\bar{N}_l(\theta,\phi)d\phi &(5.18)\\
\varepsilon(\phi) &= \bar{N}_l(\theta,\phi) - \bar{\bar{N}}_{l,\theta} &(5.19)
\end{aligned}$$

Here $\bar{N}_l(\theta,\phi)$ is written as its average over ϕ – i.e. $\bar{\bar{N}}_{l,\theta}$ – plus a quantity, $\varepsilon(\phi)$, which describes the deviation of $\bar{N}_l(\theta,\phi)$ from the average. It follows that

$$\int_{-\pi}^{\pi}\varepsilon(\phi)d\phi = 0 \tag{5.20}$$

Euler's equation for eq. 5.16 subject to the condition specified in eq. 5.20 is then

$$\frac{\partial}{\partial \varepsilon}\left(e^{-a_{\nu,\theta}\left(\bar{\bar{N}}_{l,\theta}+\varepsilon(\phi)\right)} + \lambda\varepsilon(\phi)\right) = 0 \tag{5.21}$$

where λ is a Lagrangian multiplier and is constant. This implies that

$$\begin{aligned} -a_{\nu,\theta}e^{-a_{\nu,\theta}\left(\bar{\bar{N}}_{l,\theta}+\varepsilon(\phi)\right)} + \lambda &= 0 \\ \Rightarrow \varepsilon(\phi) &= const = 0 \end{aligned} \tag{5.22}$$

Thus with respect to ϕ, the stratified case is an extremal.

To determine whether or not is is a maximal or minimal solution, consider the following example:

$$\varepsilon(\phi) = \begin{cases} 1 \;, & -\pi \leq \phi < 0 \\ -1 \;, & 0 \leq \phi < \pi \end{cases} \tag{5.23}$$

Then

$$\begin{aligned} \mathcal{I}_{\nu,\theta} &= \pi e^{a(\bar{\bar{N}}_{l,\theta}+1)} + \pi e^{a(\bar{\bar{N}}_{l,\theta}+1)} \\ &= \pi e^{a\bar{\bar{N}}}\left(e^{-1}+e\right) > 2\pi e^{a\bar{\bar{N}}} \end{aligned} \tag{5.24}$$

Thus $\varepsilon(\phi) = 0$ represents a minimal of eq. 5.15 and a maximal of eq. 5.14. Therefore, with respect to ϕ, the stratified case is a minimal of $\Lambda(\tau_0, 0)$. That it is a minimal of $\Lambda(\tau_0, x)$ for all x follows from symmetry.

The dependence of $\Lambda(\tau_0, 0)$ on structure with respect to θ does not follow easily in the plane-parallel case and so is demonstrated here for a spherical plasma.

For a spherical plasma the intensity at its centre is given by

$$\begin{aligned} I_\nu &= \frac{\beta_\nu}{4\pi}\int_{-\pi}^{\pi}\int_0^{\pi}\int_0^{R} N_u(\mathbf{r})exp\left\{-\alpha_\nu\int_0^{r} N_l(\mathbf{r}')dr'\right\}\sin\theta dr d\theta d\phi \\ &= \frac{S_\nu}{4\pi}\int_{-\pi}^{\pi}\int_0^{\pi}\left[1-e^{-\alpha_\nu R\bar{N}_l(\theta,\phi)}\right]\sin\theta d\theta \end{aligned} \tag{5.25}$$

Consider the integral

$$\begin{aligned} \mathcal{I}_{\nu,\phi} &= \int_0^{\pi} e^{-\alpha_\nu \bar{N}_l(\theta,\phi)R}\sin\theta d\theta \\ &= \int_0^{\pi} e^{-b_{\nu,\phi}\left(\bar{\bar{N}}_{l,\phi}+\eta(\theta)\right)}\sin\theta d\theta \end{aligned} \tag{5.26}$$

where

$$b_{\nu,\phi} = -\alpha_\nu R \tag{5.27}$$

$$\bar{\bar{N}}_{l,\phi} = \int_0^\pi \bar{N}_l(\theta,\phi)\sin\theta d\theta \tag{5.28}$$

$$\eta(\theta) = \bar{N}_l(\theta,\phi) - \bar{\bar{N}}_{l,\phi} \tag{5.29}$$

Hence

$$\int_0^\pi \eta(\theta)\sin\theta d\theta = 0 \tag{5.30}$$

Euler's equation for eq. 5.26 subject to the condition specified in eq. 5.30 is then

$$\begin{aligned} \frac{\partial}{\partial\eta}\left(e^{-b_{\nu,\phi}\left(\bar{\bar{N}}_{l,\phi}+\eta(\theta)\right)}\sin\theta + \lambda\eta(\theta)sin\theta\right) &= 0 \\ \Rightarrow -b_{\nu,\theta}e^{-b_{\nu,\theta}\left(\bar{\bar{N}}_{l,\phi}+\eta(\theta)\right)}\sin\theta + \lambda\sin\theta &= 0 \\ \Rightarrow \eta(\theta) &= const = 0 \end{aligned} \tag{5.31}$$

Thus with respect to θ, the stratified case is an extremal. Moreover, as before, with respect to θ, the stratified case represents a minimal of $\Lambda(\tau_0, 0)$.

Discussion

It follows from the above that it is possible to have two plasmas of the same apparent optical depth in terms of emission, but different optical depths from the perspective of absorption. This might be observable in disk centre spectra. Consider the intensity ratio of the 2–2 to the 1–2 component of the C III $2s2p\ ^3P$ – $2p^2\ ^3P$ multiplet ($\sim$ 1175 Å). Since these two lines arise from a common upper level, this ratio is proportional to that of their $\bar{g}\{\tau_0\}$ values which are determined by their line-of-sight optical depths. From an observation of this ratio at disk centre the optical depths of these lines may be deduced and from them the optical depths of all the other lines of C III may also be calculated. Thus all the C III optical depths may be known from a single $\bar{g}\{\tau_0\}$ ratio (see chapter 2). Now consider the intensity ratio of the 2–1 to the 1–2 component of the same C III multiplet. These lines do not share an upper level but have almost identical optical depths. Thus this ratio is proportional to that of their upper level population densities which is determined by their $\bar{\bar{g}}\{\tau_0/2\}$ values. These in turn are dependent specifically upon optical depths at disk centre and thus the optical depths of all the C III may be deduced from the I(2–1)/I(1–2) intensity ratio. If the plasma is stratified then the two sets of optical depth should be in agreement. If it is not then those inferred from the latter ratio will be lower than those of the former.

This reasoning follows from consideration of two plasmas of the same total number of particles. If, on the other hand, a plasma is de-homogenised by the removal rather than re-organising or particles – that is, so that the two plasmas are not of the same total particle number – then the same conclusion results. Removing plasma so as to leave spicule like structures, for example, means that at disk centre the optical depths are the same but the degree of absorption is reduced.

The latter perspective is more useful when considering spicule-like structures (i.e. radial structures) at disk centre. The former perspective (re-organisation rather than removal) is more useful when considering such structures at the limb.

5.2 Models with flow

Flow introduces a dependence of the absorption and emission profiles on position. In the zero-flow case the absorption and emission profiles share the same centroid location and so when flow is introduced it can only act so as to displace one profile with respect to the other. Only moderate opacities, where the source function is close to constant, are considered here. In this regime, and in the absence of line blending, opacity serves at most to influence line profiles by way of flattening their peaks. In more severely thick circumstances partial frequency redistribution, which leads to a dependence of the source function upon frequency, can lead to self-reversal of spectral lines (see fig. 1.8). Thus for the optical depth regime considered here, if there is no line blending the only effect of this displacement is to reduce the degree of absorption. Thus the zero-flow, stratified case represents a minimal of $\Lambda(\tau_0, x)$. This may be illustrated as follows: consider the intensity, I_r, at the point 0 due to emission along a ray path of length R. This is given by

$$I_r \sim \int_0^R \int_0^\infty \phi_a(\nu,0)\phi_e(\nu,r)exp\left\{-\int_0^r \kappa_0(r')\phi_e(\nu,r')dr'\right\} dudr \tag{5.32}$$

where $\phi_a(\nu,r)$ and $\phi_e(\nu,r)$ respectively denote the absorption and emission profiles at the point r. For Doppler broadened lines eq. 5.32 may be written as

$$I_r \sim \int_0^R \int_{-\infty}^\infty e^{u^2} e^{(u+\eta(r))^2} exp\left\{-\int_0^r \kappa_0(r') e^{(u+\eta(r'))^2} dr'\right\} dudr \tag{5.33}$$

where $\eta(r)$ is some function describing the Doppler shift of the line profile due to the flow velocity at the point r. If $f(\kappa_0,\eta,r)$ is defined such that

$$I_r \sim \int_0^R f(\kappa_0,\eta,r)dr \tag{5.34}$$

then for extremals with respect to η, Euler's equation is

$$\begin{aligned} &\frac{\partial f}{\partial \eta} = 0 \\ \Rightarrow &\int_{-\infty}^\infty \Bigg[e^{u^2} e^{(u+\eta(r))^2} (-2(u+\eta(r)))exp\left\{-\int_0^r \kappa_0(r') e^{(u+\eta(r'))^2} dr'\right\} \\ &\quad + e^{u^2} e^{(u+\eta(r))^2} exp\left\{-\int_0^r \kappa_0(r') e^{(u+\eta(r'))^2} dr'\right\} \\ &\quad \times \int_0^r \kappa_0(r') e^{(u+\eta(r'))^2} 2(u+\eta(r'))dr' \Bigg] du = 0 \end{aligned} \tag{5.35}$$

This is satisfied if $\eta(r) = 0$ for all values of r and thus this condition specifies an extremal. To determine whether or not it is a maximal or minimal, consider the case where at all points along the ray the plasma is stationary except at the point of absorption ($r = 0$) where there is flow parallel to the ray. In this case the line profile due to emission along the ray is displaced in frequency space from the absorption profile at $r = 0$. The convolution of the two inevitably results in a decrease in absorption compared to the zero-flow case. Thus when there is no line blending the zero-flow case represents a minimal of the absorption factor.

In the case of line blending the situation becomes more complex. Blending itself leads to a decrease in the absorption factor and so since flow will contribute to the extent of blending, a *decrease* in the absorption factor results. Thus when there is blending, the zero-flow case does not necessarily represent a minimal.

In general flow and blending ought to be treated self-consistently but if the main influence of flow is to broaden emission profiles then flow may be accounted for within the blending formulation. In this event flow is included implicitly. This has in fact already been the case in the analyses presented so far since spectral lines emanating from the TR have Gaussian profiles broadened beyond their thermal widths due to non-thermal velocities (Spadaro et al., 1996).

5.3 Concluding remarks

The escape probability approach hinges on the separability of the effects of opacity upon the population structure and on emergent fluxes. This separability de-couples and linearises the equations of radiative transfer and statistical balance. In a stratified, stationary atmosphere an optical depth regime exists whereby this separability is possible and where opacity effects are significant and observable. This regime is defined according to the degree of absorption within the plasma and the extent of the modification to the upper level population density distribution due to opacity which are characterised by the absorption factor, $\Lambda(\tau_0, x)$. Following this the influence of structure and flow upon the extent of this optical depth regime must be addressed. Both structure and flow serve to diminish the degree of absorption as compared with the stratified, stationary case, except when there is blending where flow can enhance the blending effects and thus lead to an increase in the degree of absorption. Providing that plasma flow influences emission profiles by way of broadening and does not distort their shape then flow may be accounted for within blended lines as well as unblended ones. It is therefore appropriate to consider the stratified, stationary atmosphere in determining the region of validity of the $\bar{g}\{\tau_0\}$ quantity as structure and flow will not further restrict this region.