

# Modified Relativistic Approach for Atomic Data Calculation

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# Outlook

- 1 Relativistic integral analogues
  - Method
  - Changes to standard R-matrix codes
  - Results
- 2 Quasirelativistic Hartree-Fock Approach
  - Method
  - Energy Level Spectra
  - Transition Line Spectra



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# Relativistic Integral Analogues

- A relation for multipole integral

$$M^k(n_1 l_1, n_2 l_2) = \frac{1}{2} \sum_{j_1 j_2} [j_1, j_2] \left\{ \begin{matrix} j_1 & j_2 & k \\ l_2 & l_1 & 1/2 \end{matrix} \right\}^2 M^k(n_1 l_1 j_1, n_2 l_2 j_2)$$

- A non-relativistic multipole integral

$$M^k(n_1 l_1, n_2 l_2) = \int_0^\infty dr P_{n_1 l_1} r^k P_{n_2 l_2}$$

- A relativistic multipole integral

$$M^k(n_1 l_1 j_1, n_2 l_2 j_2) = \int_0^\infty dr r^k \left[ P_{n_1 l_1 j_1} P_{n_2 l_2 j_2} + Q_{n_1 \bar{l}_1 j_1} Q_{n_2 \bar{l}_2 j_2} \right]$$



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# Changes in standard R-matrix codes

## Amendments to R-matrix flow

- Start from relativistic R-matrix version (DARC)  
GRASP → DSTG0 → DSTG1/ORB/INT
- Follow with non-relativistic R-matrix (RmaX)  
AUTOSTRUCTURE → STG1 → STG2 → STGH
- Finish with intermediate coupling frame transformation  
method (ICFT)  
STGICF → STGF
- Test on electron-impact excitation of  $2s^2 - 2s2p\ ^3S^1$   
transition

*V.Jonauskas et al., J.Phys B* **38** (2005) L79-L85



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# Outlook

## 1 Relativistic integral analogues

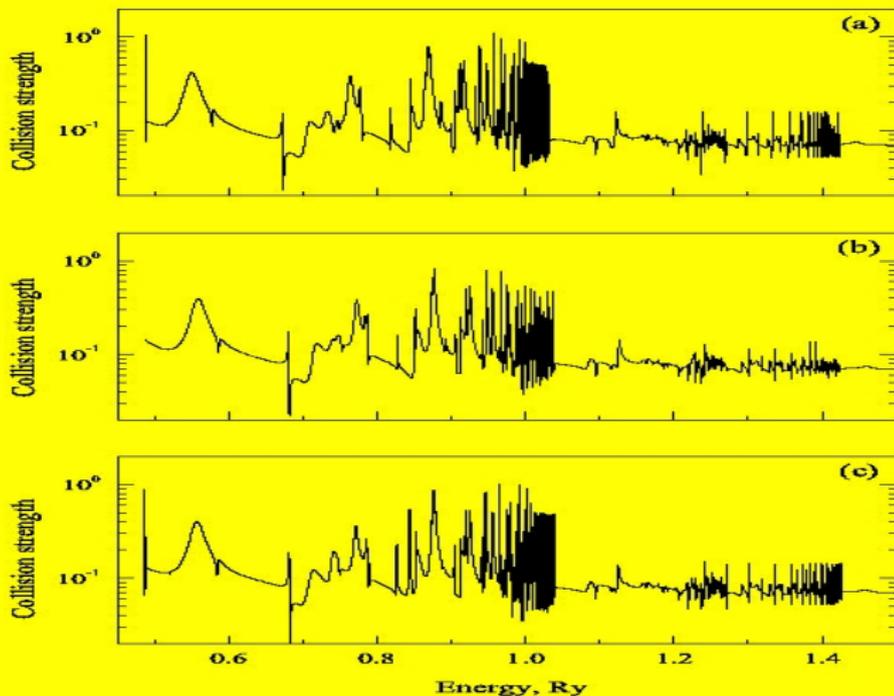
- Method
- Changes to standard R-matrix codes
- **Results**

## 2 Quasirelativistic Hartree-Fock Approach

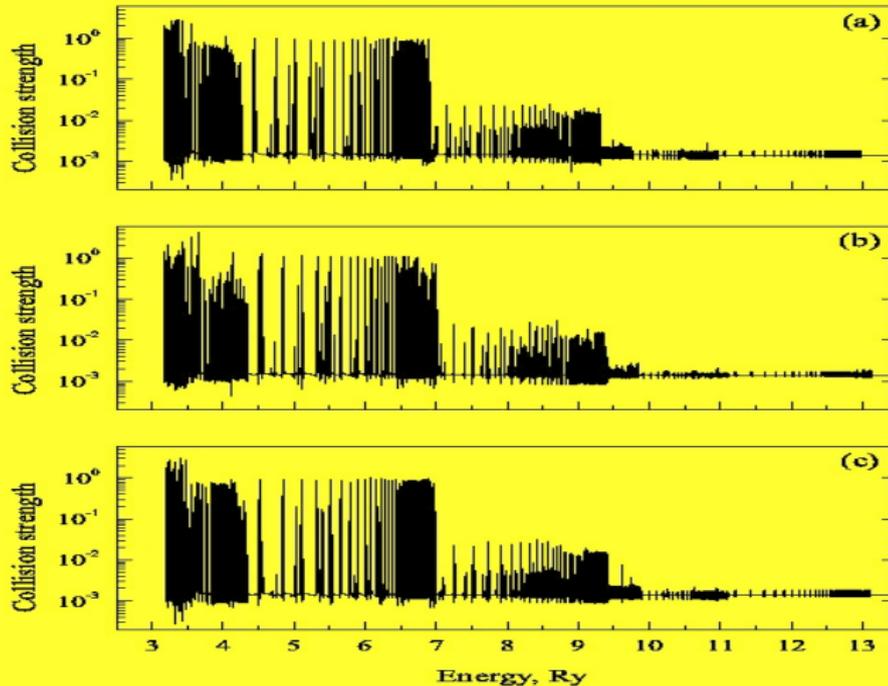
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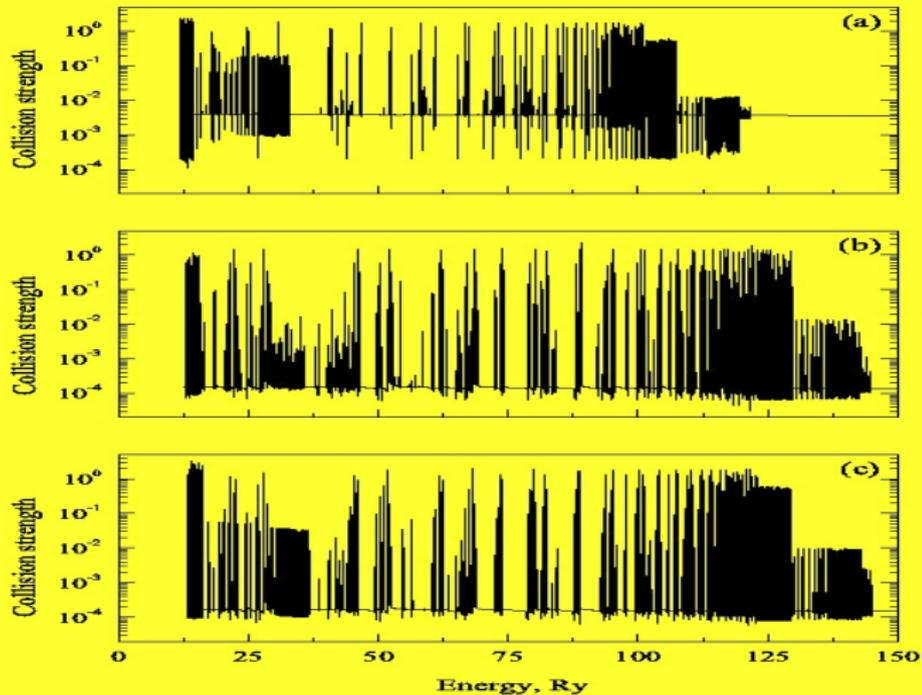
# Electron-impact excitation $C^{2+}$



# Electron-impact excitation $\text{Fe}^{22+}$



# Electron-impact excitation $W^{70+}$



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# General form of QRHF equations

$$\begin{aligned}
 & \left( \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - V(nl|r) - \epsilon_{nl} \right) P(nl|r) - X(nl|r) + \text{non-relativistic terms} \\
 & \frac{\alpha^2}{4} (\epsilon_{nl} + V(nl|r))^2 P(nl|r) + \frac{\alpha^2}{4} (\epsilon_{nl} + V(nl|r)) X(nl|r) + \text{mass-velocity terms} \\
 & \frac{\alpha^2}{4} \left( \delta_{l,0} + \frac{1}{3} \delta_{l,1} \right) \frac{\frac{dU(r)}{dr} \left( \frac{d}{dr} - \frac{1}{r} \left( \alpha^2 Z^2 \delta_{l,1} \left( -\frac{37}{30} - \frac{5}{9n} + \frac{2}{3n^2} \right) + 1 \right) \right)}{1 - \frac{\alpha^2}{4} (\epsilon_{nl} + V(nl|r))} P(nl|r) = 0 \quad \text{contact term}
 \end{aligned}$$

# Main Distinctions

- No statistical potentials are used. Only conventional self-consistent filed direct  $V(nl|r)$  and exchange  $X(nl|r)$  potentials in **QRHF**
- The finite size of nucleus is considered determining potential  $U(r)$
- The mass-velocity term splits into two parts
- No two-electron potentials in the numerator
- Only direct part of  $V(nl|r)$  in denominator of the contact interaction
- Contact interaction with nucleus is defined both for  $s$ -electrons and  $p$ -electrons



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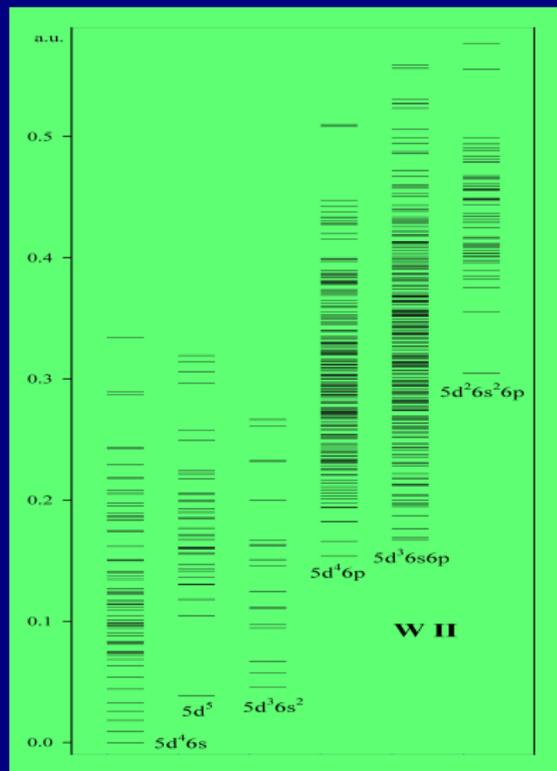
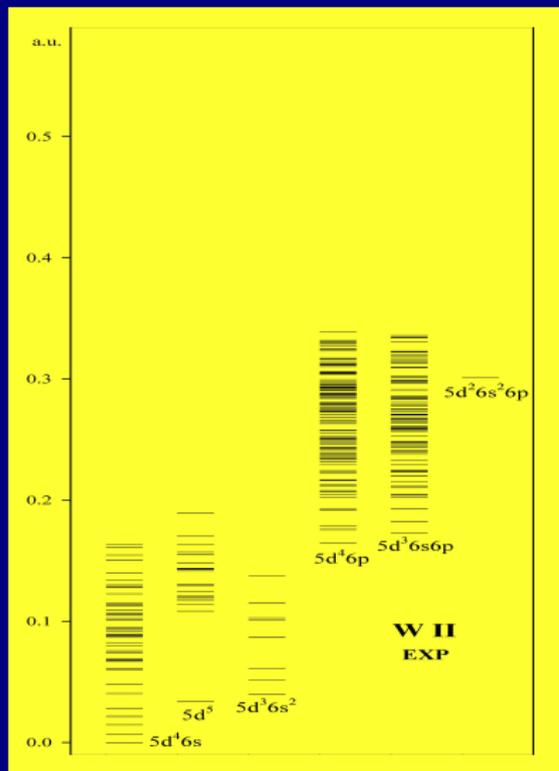


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# Energy Levels for W II

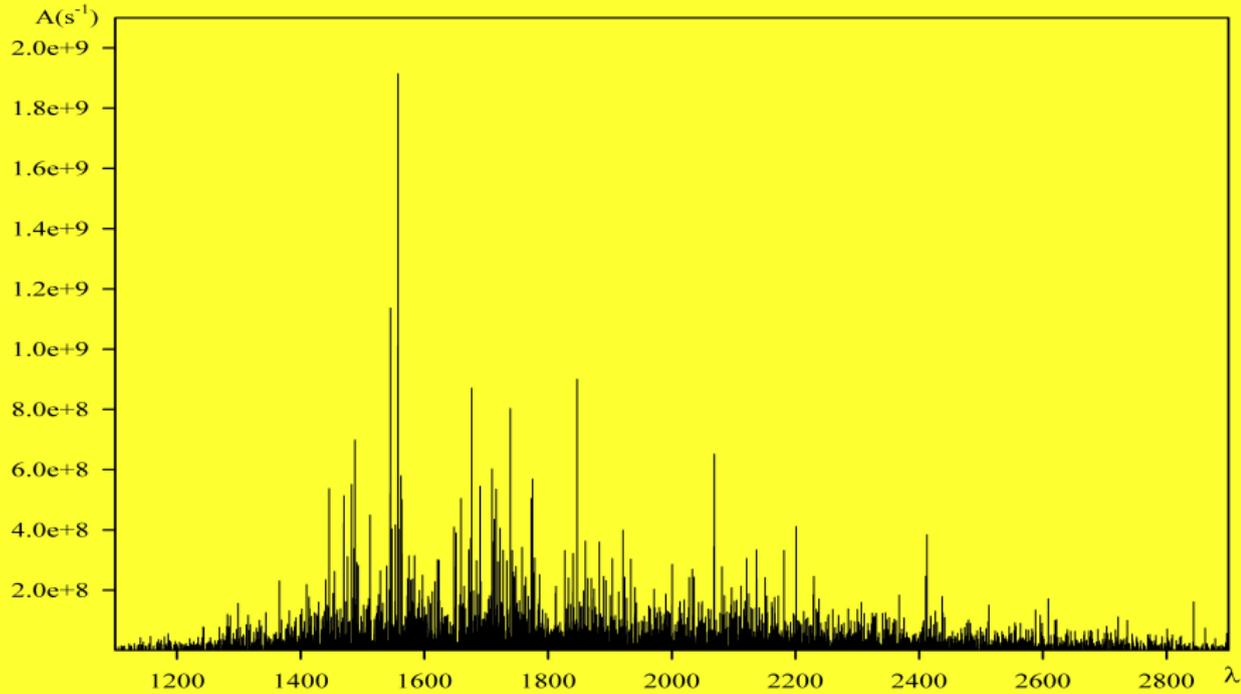


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# Transition probabilities for W II



# Summary

- Method of Relativistic Integral Analogues (ARI) for electron scattering calculation
- Quasirelativistic Hartree-Fock (QRHF) approach for discrete spectra

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# THANK YOU

