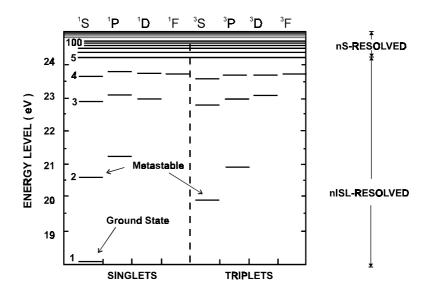
# ADAS311: Beam emission spectroscopy - process beam stopping and emission : He-beam

The program calculates the quasi-static excited population structure, collisional-radiative cross coupling and recombination coefficients of helium atoms in an impure plasma. A bundle-nISL approximation is adopted for low lying levels while for higher lying levels, where l-redistribution is complete, a bundle-nS approach is used. The helium atoms may be part of the thermal plasma or may be in a beam. The latter case is the only one of relevance for this manual, however the full flexibility of the program has been retained.

# **Background theory:**

ADAS311 has been designed to operate in a similar manner to the bundle-nS deuterium beam model, ADAS310. A bundle-nlSL treatment is employed to calculate the excited population structure of helium atoms either in a optically thin thermal plasma or in a mono-energetic beam. It is an nl-spin resolved model which calculates the populations of the l-substates from the ground state up to an arbitrary principal quantum number, above which a bundled-nS treatment is then adopted, see figure 4.11.1.



**Figure 4.11.1** Schematic illustration of the bundle-nlSL model. The low levels are calculated in an nl-resolved picture up to an arbitrary principal quantum shell, above which a bundle-nS treatment is adopted.

The switching between the bundle-nISL and the bundle-nS treatment is such that for higher principal quantum shells 1-redistribution is effectively complete. The ground state and the He(2 ¹S) and He(2 ³S) metastables are treated as non-equilibrium levels, therefore the equilibrium populations of the excited states are calculated relative to each non-equilibrium level. The statistical balance equations are similar to those of ADAS310 but contain additional processes which were not applicable in the bundle-nS picture. These include spin changing electron collisions and collisional transitions between degenerate levels.

#### Radiative processes

The approximate methods employed to evaluate the rate coefficients for the radiative processes in the bundle-nS approximation of ADAS310 are applicable for non-hydrogenic ions and are used in ADAS311. Where appropriate however exact energy levels and statistical weights for helium are adopted. Exact energy levels for the low levels in the bundle-nlSL treatment are expanded over the complete manifold of levels using the quantum defect method for each spin system. That is the quantum defect is calculated using a series expansion.

$$\mu = a_1 + a_2 E_1 + a_3 E_2^2 + a_4 E_3^3$$
 4.11.1

where the energy levels  $E_1$ ,  $E_2$  and  $E_3$  are the exact values. This is repeated for the s, p, and d l-series and for both spin systems with the energy levels then calculated from,

$$E_{nl} = \frac{z_0^2 I_H}{(n-\mu)^2} \tag{4.11.2}$$

Higher l-series have negligible quantum defects. The statistical weights depend on whether we are concerned with levels which are part of the low level nlSL-resolved treatment or the high level bundle-nS picture. If we define  $n_c$  as the arbitrary principal quantum shell which separates the nlSL and nS resolved treatment. The statistical weights are defined as,

$$n < n_C$$
  $i \equiv nlSL$   $w_i = (2L+1)(2S+1)$  4.11.3

$$n > n_c$$
  $i \equiv nS$   $w_i = n^2 (2L_p + 1)(2S_p + 1)$  4.11.4

where  $L_p$  and  $S_p$  are the total angular and spin quantum numbers associated with the parent ion. We also require nl-resolved bound-bound and bound-free Gaunt factors to evaluate quantities such as the spontaneous emission and the radiative recombination coefficient. The expressions used to evaluated these factors are of considerable complexity and a detailed account can be found in the work of Summers[1].

#### Collisional processes

The expressions which were used to evaluate the collision cross sections for ADAS310 form a starting point for ADAS311. However there are several additional processes which have to be taken into account which were not applicable in the nS-resolved picture of ADAS310. These processes include ion and electron collisions between degenerate levels and electron driven spin changing collisions. The approximate methods to evaluate the rate coefficients for each of these processes is discussed in detail by Summers[1]. For degenerate collisions due to electrons and ions, the rate coefficient is given as,

$$q_{nlSL \to n'l'SL'} = \sqrt{\pi} \alpha c a_0^2 \left(\frac{I_H}{kT_e}\right)^{\frac{1}{2}} \left(\frac{m}{m_e}\right)^{\frac{1}{2}} \frac{1}{z_1^2} D_{nlSL \to n'l'SL'} \left\{ 26.57 + \log \left(\frac{z_1^2 T m_e}{D_{nl} m}\right) + \log R_c^2 \right\}$$

where m and T are respectively the mass and temperature of the colliding particle. The quantity  $R_c^2$  is described as a cut-off to ensure the collision cross sections have finite values. The expression for spin changing transitions between each spin system is,

$$q_{nlSL \to n'l'S'L'} = \frac{5}{12} \frac{(2L'+1)(2S'+1)}{(2L_p+1)(2S_p+1)} \left| \eta_{n'l'}^{nl} \right|^2 q \left( \varepsilon_1 \to \varepsilon_2 \right) \qquad S \neq S' \quad 4.11.5$$

where  $\eta^{nl}_{\phantom{n}n'l'}$  is an overlap fractional function.

#### b-factors and scaling

It is convenient to rewrite the statistical balance equations in terms of the deviations of the populations from those which would occur in thermodynamic equilibrium. The deviation  $b_i$  is defined by

$$N_{i} = N_{e} N_{+} 8 \left( \frac{\pi a_{0}^{2} I_{H}}{k T_{e}} \right)^{3/2} \frac{w_{i}}{2w_{+}} \exp(I_{i} / k T_{e}) b_{i}$$
 4.11.6

Also it is convenient to introduce  $c_i = b_i - 1$  and scaled temperatures and densities

$$t_e = \left(\frac{kT_e}{I_H}\right) \frac{1}{z_0^2}, \ t_r = \left(\frac{kT_r}{I_H}\right) \frac{1}{z_0^2} \text{ and } \rho_e = 2^5 \sqrt{\frac{\pi}{3}} \frac{\pi a_0^3}{\alpha^3} \frac{N_e}{z_0^7}$$
 4.11.7

with similar forms for the proton temperature and density. In these terms the statistical balance equations become particularly suitable for calculation.

#### Solution of the collisional-radiative equations

The general solution of the statistical balance equations to obtain collisional-radiative coefficients and populations is described in detail in the description of ADAS208. methods of condensation, projection and expansion which gives flexibility in the treatment of metastables are also described there. For the case of helium in beams entering fusion plasmas, the bundle-nISL level of resolution is appropriate for low lying levels which are of spectroscopic interest, while for highly excited states the bundle-nS treatment is suffice. The cross coupling coefficients evaluated by ADAS311 describe the rate at which the ground state and the He(2 <sup>1</sup>S) and the He(2 <sup>3</sup>S) metastables are populated and depopulated within a collisional-radiative frame work. They include the influence of stepwise atomic processes and are used to model the attenuation of a fast neutral helium beam, see description of ADAS313. Nonetheless, in the case of a slow helium beam where the excited levels do not reach equilibrium, it is necessary to project the influence of the highly excited quasi-static levels onto the low lying non-equilibrium levels. This results in a series of cross coupling coefficients which connect all the excited levels which are of spectroscopic interest. These coupling coefficients are then used to assemble a series of equation which are solved to obtain the nonequilibrium population of each low lying level. ADAS311 has some capability for supporting such studies which it does by accessing an 'expansion definition file' and by archiving the 'condensed-projection matrices'. These aspects have been de-activated for the present version of ADAS311 although the expansion definition file is still requested.

#### Supplementary precision data

The general purpose routines described above and in the description for ADAS310 are satisfactory for generating collision cross-section data at moderate accuracy for the bundle-nlSL atom population structure. However the precision may be improved for excitation, ionisation and charge transfer involving the ground level and possibly other low lying l-substates. Such improvement is particularly important for cross coupling coefficients and active or passive Lyman and Balmer line spectroscopy. Systematic improvement is implemented by drawing on the ADAS ion/atom and ion/electron collision database. A supplementation subroutine (NSUPHE1 for He<sup>0</sup> (HeI) ), which is subject to periodic adjustment executes the data acquisition under the following classes

class	returned data	source
electron excit.	excit. & dexcit. maxwellian. rate coefficients	specific ion files ( <i>adf04</i> ) bundle-nl form
electron ions.	ionis. maxwellian rate coefficient	szd files (adf07)
proton excit.	excit. & dexcit. displaced maxwell. rate coefficients	sia#he files (adf02)
proton ionis.	ionis displaced maxwell. rate coefficient	sia#he files (adf02)

impurity excit. excit. & dexcit. displaced sia#he files (adf02)

maxwell. rate coefficients

impurity ionis. ionis.. displaced sia#he files (adf02)

maxwell. rate coefficient

The supplementary data is introduced, for example as an effective P factor

$$\begin{split} P_{i,i'} &= q_{i \to i'}^{(s)} / \{ (2\sqrt{\pi}\alpha \, ca_0^2) \\ & \sqrt{I_H / kT} [ (I_H / \Delta E_{i,i'}) f_{i \to i'}] \exp(-\Delta E_{i,i'} / kT) ] \} \end{split} \tag{4.11.8}$$

for excitation which replaces the moderate accuracy formulaic value if available. For the hydrogenic ion as a plasma species, state selective charge exchange recombination data to excited levels for helium donors may be included and these data are accessed directly from the appropriate file (*adf01*). In the present implementation of ADAS311 this has also been deactivated even though the input file is still required.

#### Distinctions in analysis for helium in beam or plasma

The program can apply to helium atoms in a thermal plasma or to helium atoms in a beam. The practical distinction is made by the assignment of a translational velocity for beam atoms. This velocity is incorporated in the integrals of beam particle / plasma particle cross-sections over the Maxwellian distributions in the thermal plasma. For helium forming part of the thermal plasma, the translational velocity is set to zero. In the latter circumstance, ion impact collision rates are very small compared with electron impact rates. Also recombination (both free-electron capture and charge exchange capture) become significant processes. For the helium atoms in a fast beam, recombination is not relevant and although formally present is ignored in the results. However the translational velocity can make ion impact collisions more important than electron collisions.

#### Multiple impurities and $z_{eff}$

In general several impurity species (nuclei) may be involved in collisionally inducing transitions. Of these the hydrogen nuclei are often the most important and because of better availability of data and approximations for proton induced rates, the protons are usually treated as a special (plasma) species and the other species treated as the 'impurities'. Let the electron and proton number densities be  $N_e$  and  $N_p$  respectively. Let the set of impurities charges and fractions by number be  $\{z_{0_i}^{(imp)}, f_i^{(imp)}; i=1,...,I\}$  then the total impurity number density,  $z_{eff}$  and component impurity number densities are

$$N^{(imp)} = (N_e - N_p) / (\sum_{i=1}^{I} z_{0_i}^{(imp)} f_i^{(imp)})$$
 4.11.11

$$z_{eff} = (N_p + N^{(imp)} [\sum_{i=1}^{I} (z_{0_i}^{(imp)})^2 f_i^{(imp)}]) / N_e$$
 4.11.12

$$N_i^{(imp)} = f_i^{(imp)} N^{(imp)}$$
 4.11.13

In the case of single impurity, which is frequently used as a single 'effective' impurity, it is convenient to alter the definitions. Taking  $N_e$ ,  $N_p$  and  $z_{eff}$  as prescribed

$$z_0^{(imp)} = (z_{eff} N_e - N_p) / (N_e - N_p)$$
 4.11.14

$$N^{(imp)} = (N_e - N_p) / z_0^{(imp)}$$
 4.11.15

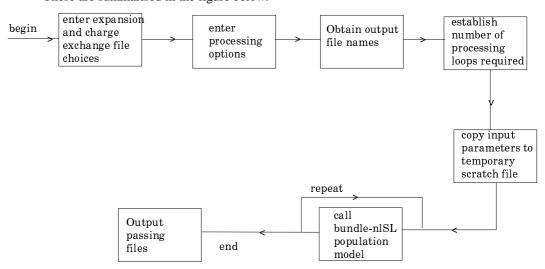
determines the effective charge for the single impurity and its number density. As mentioned earlier, the collisional-radiative cross coupling coefficients are employed to model the attenuation of a helium beam. However there is sometimes confusion about their definition. Although hydrogen nuclei impact ionisation and charge transfer are the most efficient reactions causing beam stopping, it is usual in fusion to define the coupling coefficient with respect to the electron density. For ADAS311, this applies even if the coupling coefficients are calculated for the case of a pure impurity with all the free electrons contributed by the impurity in charge balance.

#### Population results and preparing tabulations

ADAS 311 is too slow in execution for a direct link to inter-pulse experiment analysis and so it is used to prepare tabulations of effective cross coupling and beam emission coefficients for subsequent look-up. In the case of a fast beam, the effective coefficients are most sensitive to the beam particle energy and the plasma ion density and less sensitive to plasma ion temperature and Z-effective. Suitable tabulations can therefore be built on a reference set of plasma and beam conditions, a two-dimensional array of coefficients as functions of beam energy and plasma density at the reference conditions of the other parameters and then onedimensional vectors of the coefficients as functions of each minor parameter at the reference condition of all the other parameters. ADAS311 accepts as input the definition of these scans, establishes an extended list of cases required to achieve these scans and then executes repeated population calculations at each set of plasma conditions in the list. ADAS311 can compute the populations for any mixture of light impurities (hydrogen to neon) in the plasma. It is impractical to deal with all possible mixtures of impurities. It is our usual practice to execute ADAS311 in turn for each light impurity from hydrogen to neon treated as a pure species. The mixed species effective coefficients are constructed from these pure impurity solutions by the theoretical data acquisition routines. The main population output is very complete and in principle contains all information on possible emitted spectrum lines up to very high n-shells together with both cross coupling and recombination collisional-radiative coefficients. It is archived as ADAS data format adf26. However the actual assembly of the cross coupling and beam emission coefficients into their respective ADAS data formats of adf21 and adf22 is carried out using ADAS313.

# **Program steps:**

These are summarised in the figure below.



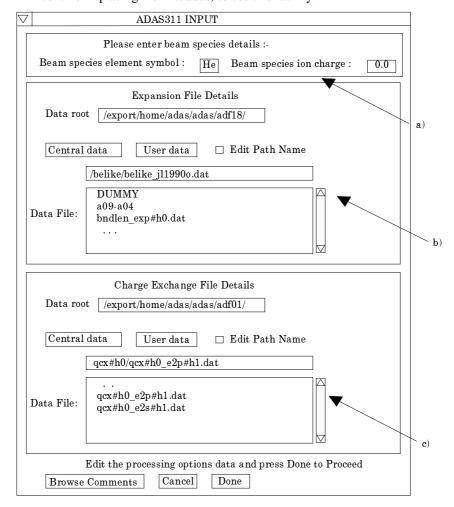
# Interactive parameter comments:

The program which makes use of data from archived ADAS data-sets initiates an interactive dialogue with the user in three parts, namely, input data file selection, entry of user data and disposition of output. The primary calculations may be conducted in foreground or background.

Move to the directory in which you wish the ADAS created files to appear. These include the output text file produced after executing any ADAS program (*paper.txt* is the default), and up to two further output data files. There is no graphical output from this program Initiate ADAS311 from the program selection menus in the usual manner.

#### The **file selection window** appears first as illustrated below.

- 1. At a) enter the beam species (He for helium and its isotopes) and the atomic charge of the beam species. Only data for neutral beam species is present in the central ADAS database at this time.
- 2. There are two data files to be selected, the expansion file and the charge exchange file. The procedure is the same in both cases.
- 3. Data root b) shows the full pathway to the appropriate data sub-directories. Click the *Central Data* button to insert the default central ADAS pathway to the correct data type. Note that each type of data is stored according to its ADAS data format (*adf* number). Click the *User Data* button to insert the pathway to your own data. Note that your data must be held in a similar file structure to central ADAS, but with your identifier replacing the first *adas*, to use this facility.

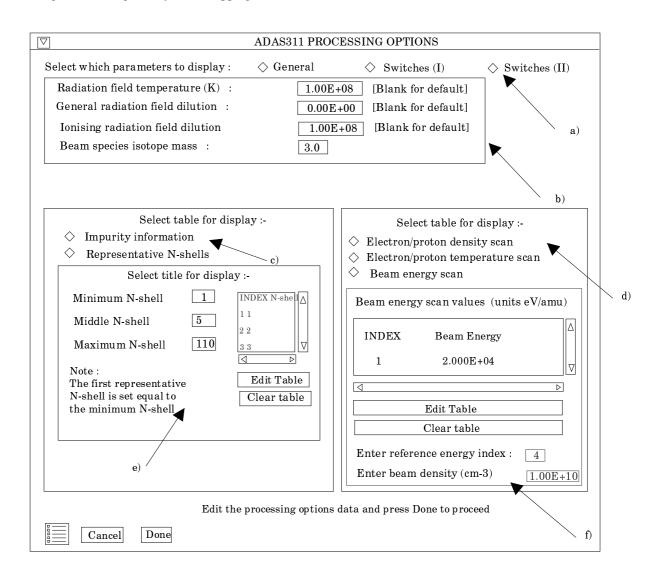


- 4. The Data root can be edited directly. Click the *Edit Path Name* button first to permit editing
- 5. Available sub-directories are shown in the large file display window b). Scroll bars appear if the number of entries exceed the file display window size.
- 6. Click on a name to select it. The selected name appears in the smaller selection window c) above the file display window. Then its sub-directories in turn are displayed in the file display window. Ultimately the individual data-files are presented for selection. Data-files all have the termination .dat.
- 7. The expansion file is selected at b). A special ADAS data type *adf18* is used for such 'expansion' and 'cross-referencing' files. They fall into various categories, kept in

- sub-directories, according to where they map from and to. For the moment note that <code>bndlen\_exp#he0.dat</code> is the one needed here and it sits alone as shown in the illustration. Always select it.
- 8. The charge exchange file is selected at c). Again this is not of importance for neutral beam stopping. The charge exchange data set is required when hydrogen nuclei can act as electron receivers from other species. You will see no effect of your selection here on the beam stopping coefficient but the selection is kept in for the future. Once a charge exchange data file is selected, the set of buttons at the bottom of the main window become active.
- 9. Clicking on the *Browse Comments* button displays any information stored in the selected charge exchange data-file. It is important to use this facility to find out what is broadly available in the data-set. The possibility of browsing the comments appears in the subsequent main window also.
- 10. Clicking the *Done* button moves you forward to the next window. Clicking the *Cancel* button takes you back to the previous window

### The processing options window has the appearance shown below

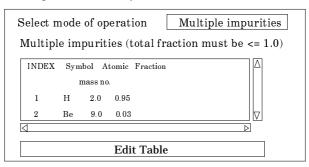
11. The various control parameters of the collisional-radiative population calculation are organised into three groups selected in turn by the buttons *General*, *Switches (I)* and *Switches (II)*. These cause the appropriate set of parameters to be displayed at b). The default settings are reasonable and they can be ignored as long as only beam stopping is the intent.



12. Switches (I) allow some choices to do with electron collision cross-sections and Switches (II) allow some choices to do with ion collision cross-sections. (see the corresponding sub-window inserts below Make sure that Access to low level data is chosen at a) and Use beam energy informing cross-sections at b). It is this latter piece of information that informs the calculation that the neutral hydrogen is in the beam and not in the plasma.

Please enter the following parameter switches:_			
Delta N range for impact parameter cross-sections	: 2 [max value of 4]		
Integral order for impact parameter cross-sections	: [3 [max value of 3]		
Use Percival-Richards cross-sections :	NO [NO defaults to van Regemorter X-section]		
Access special low level cross-section data :	YES >		
Please enter the following parameter switches :_			
Activate ion impact cross-sections :	[max value of 4]		
Delta N range for ion impact cross-sections :	[max value of 3]		
Use Lodge ion impact cross-sections :	NO [NO defaults to Vainshtein X-section]		
Use beam energy in forming ion cross-sections :	YES		
	h		

- 13. The choices at c) above must then be made. Click the *Impurity information* or *Representative N-shell* buttons. The appropriate sub-window is displayed at c).
- 14. The representative N-shells requires specification of the lowest N-shell, middle N-shell and a Highest N-shell as well as a set of sensibly spaced 'representative' N-shells spanning the range. It should be noted that the middle N-shell distingiushes between the bundle-nlSL and bundle-nS tretament. The population of all the angular substates from the lowest to the middled N-shell are evaluated, while above the middle N-shell only the population of the principal quantum shells are evaluated. Make sure the lowest is 1 for helium and the middled N-shell is 5. Make the highest around 110 and use about 20 representative levels. Use all levels up to N=10 and then start to space more widely.



- 15. The Impurity information sub-window is shown below. There are two modes of operation. Single impurity or Multiple impurities. Click the drop-down list button to make your choice.
- 16. The single impurity case has only one impurity nucleus in addition to protons present in the plasma. The single impurity case is used to build up such data sets in *adf21* and *adf22* type format, see description for ADAS313. Note how the impurity and protons fit together.
- 17. The multiple impurity choice enables us to investigate the influence of an impurity mixture on the coupling coefficients with greater precision. Edit in the fractions you wish in the usual manner. Note that the impurity density acts non-linearly in the

coupling coefficient and so the linear superposition implied by the use of ADAS304 is imprecise. It is however very fast which is necessary in large scale experimental data analysis.

- 18. At d) the choice of plasma and beam parameters for the scans are made Click on the appropriate button to work on each scan in turn. Note that you edit in a set of values and then choose one to be the reference value of that parameter. The table may be edited by clicking on the *Edit Table* button.. The ADAS Table Editor window is then presented with the same set of editing operations available as are described in bulletin *nov18-94*. Values should be monotonic increasing. It has proved helpful to add a *Clear Table* button to remove all entries in the output field.
- 19. Note that a neutral helium density in the beam is requested. This is necessary to allow a mathematical separation of the various influences on the neutral helium population structure and is not an experimental beam density. A value of order 10<sup>6</sup> or greater is suitable for the program operation.
- 20. Clicking the *Done* button causes the output options window to be displayed. Remember that *Cancel* takes you back to the previous window.

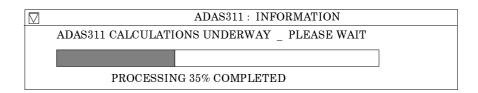
#### The **output options window** is shown below.

21. It follows the usual pattern except that there is no graphical output.

The Run Summary is given in the output file specified at a). The Run Summary Output button activates writing to a text output file. The file name may be entered in the editable File name box when Run Summary Output is on. If the file already exits a choice to Append or Replace may be made. The default file name 'paper.txt' may be set by pressing the button Default file name. A 'pop-up' window issues a warning if the file already exists and the Append or Replace button has not been activated.

$\nabla$	ADAS311 OUTPUT OPTIONS	
	Title for run:	
	□ Run Summary Output □ Append □ Replace □ Default file name	
	File name paper.txt	
	☐ First passing file ☐ Append ☐ Replace ☐ Default file name	a
	File name data.pass1	b
	☐ Second passing file ☐ Append ☐ Replace ☐ Default file name	
	File name data.pass2	<b>c</b> )
	Cancel Run Now	

- 22. Two additional passing files may be produced. The first passing file contains line printer formatted pages of data, one page for each individual population structure case run. The data held on these sheets is very comprehensive. By appropriate choice of the parameters mentioned in the processing section above and choice of input files, helium in all its possible conditions in a fusion plasma can be obtained (beam and non-beam). The second passing files contains the projection matrices. This feature is unavailable in the present version of ADAS311.
- 23. Click the *Run Now* button to initiate the calculations. These are run in fore-ground since they are of fairly modest duration. A thermometer widget illustrated below keeps you informed of the progress of the calculations.



## Illustration:

The output from the program is illustrated in table 4.11a. The tabulated output contains both the excited state population and the collisional-radiative cross coupling coefficients. A summary of the input parameters is also given.

#### Table 4.11a.

```
EFFECTIVE CONTRIBUTION TABLE FOR ION PRINCIPAL QUANTUM SHELL POPULATIONS IN THERMAL PLASMA
  HELIUM
                   Z0 = 2.00E+00
                                      Z1 = 1.00E+00
 TRAD = 1.00E + 08 K
                      TE = 1.16E + 06 K
                                           TP = 1.16E + 06 K
                      NE = 1.00E+05 CM-3
                                            NP = 1.00E+00 CM-3
     W = 0.00E+00
   EH = 1.00E+00 EV/AMU NH = 1.00E+07 CM-3
                                            NH/NE = 1.00E+02 FLUX = 1.38E+13 CM-2 SEC-1
OCOLLISIONAL DIELECTRONIC RATES
                 ALF EFFECTIVE IONISATION AND CROSS COUPLING RATES
              7.9081675E-15 -8.5961126E-10 2.0900639E-06
                                                        -6.5498303E-10
         31
              6.4304632E-13 -5.0658281E-10 -1.9117015E-08
                                                        2.0775860E-07
OLEVELS OF MULTIPLICITY 1
                                   F1(II)
                                                F1(III)
                                                                           B(CHECK)
                                                                                         B(ACTUAL)
                                                                F2
                                                                                                        NL/(B*N+)
                    F1(I)
0 IR
    N L LT
1 0 0
                9.4362731E+19 0.0000000E+00 0.0000000E+00 0.0000000E+00
                                                                         1.0737047E+15
1.0950908E+12
                                                                                       1.0737047E+15
1.0950908E+12
                                                                                                     1.0597404E-20
                              1.1597567E+20
                                            0.0000000E+00
                                                                                                     8.6224984E-21
    2 0 0
                0.0000000E+00
                                                          0.0000000E+00
                1 2865979F±07
                              3 7700009E±09
                                            5 5303704F±06
                                                           1.2068034E+02
                                                                         3 1994921F±02
                                                                                       3 1994921F±02
                                                                                                     2 5712114F-20
    3 0 0
                1.0193884E+08
                                            3.6584276E+07
                                                           2.1798098E+03
                                                                         3.6120702E+03
                                                                                       3.6120702E+03
                             1.6740420E+10
                                                                                                     8.4259830E-21
                                                                                                     2.5235828E-20
    3
                9.5648714E+06
                              4.2821961E+08
                                            4.8132494E+05
                                                           1.0442881E+02
                                                                         2.1880951E+02
                                                                                       2.1880951E+02
30 100 0 0
               9.2732992E + 03 \\ \phantom{0}1.2002970E + 05 \\ \phantom{0}4.5014386E - 05 \\ \phantom{0}4.4865574E + 00 \\ \phantom{0}4.5932069E + 00 \\ \phantom{0}4.5932069E + 00
                                                                                                     8.2867359E-17
OLEVELS OF MULTIPLICITY 3
0 IR N L L'
31 2 0 0
                                                                           B(CHECK)
                                                                                         B(ACTUAL)
                0.0000000E+00 0.0000000E+00 3.8351874E+19 0.000000E+00 1.1980260E+14
                                                                                      1.1980260E+14
                                                                                                     2.6074345E-20
                3.6114582E+07
                             2.1225959E+09 1.3754551E+11 5.2880585E+04
                                                                        4.8297269E+05
                                                                                       4.8297269E+05
                                                                                                     7.7332539E-20
                             6.1125850E-04
               3.5531061E-05
                                          2.0290738E+05
                                                         1.9135280E+01
                                                                       1.9769117E+01 1.9769117E+01
58 90 0 0
               1.1637887E-05
                             2.1689818E-04 8.9550399E+04 8.9357444E+00 9.2154797E+00 9.2154797E+00 2.0136833E-16
59 100 0 0
                                           3.9861521E+04 4.4877036E+00 4.6122220E+00 4.6122220E+00
               3.2309871E-06
                             7.0890962E-05
                                             B = F1(I)*(N1/N+) F1(II)*(N2/N+) +F1(III)*(N3/N+) +F2
                                             N1 = POPULATION OF THE 1s2 1S METASTABLE
N2 = POPULATION OF THE 2s 1S METASTABLE
                                              N3 = POPULATION OF THE 2s 3S METASTABLE
                                              N+ = POPULATION OF GROUND STATE OF NEXT IONISATION STAGE
                                              NL = POPULATION OF RESOLVED NL QUANTUM SHELL OF ION
                                             B = SAHA-BOLTZMANN FACTOR FOR RESOLVED NL QUANTUM SHELL
                                              EH = NEUTRAL HELIUM BEAM ENERGY
                                              W = RADIATION DILUTION FACTOR
                                              Z0 = NUCLEAR CHARGE
                                             Z1 = ION CHARGE+1
```

#### **Notes:**

[1] Summers, H. P. (1977) Mon. Not. R. Astr. Soc.